A Tabu Search Algorithm for a Pricing Problem on a Transportation Network

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1 Introduction

A conjunction of several factors (increased competition in a globalized economy, real-time access to a wealth of transparent information, the rise of a more knowledgeable and pragmatic generation of consumers) is currently changing the perception and nature of optimal pricing. Whereas pricing models have in the past often been approached from a purely academic standpoint, optimal pricing is nowadays considered as a central financial and operational tool in major industries, and has become one of the most important levers towards profitability.

Until recently, a key factor in pricing and revenue management problems concerned demand modelling and forecasting. However as mentioned by Van Ryzin and Liu [9], the traditional "independent demand model", which assumes that demand for each product is driven by a stochastic process that is unaffected by the availability of other products, may be ill-suited. Indeed, it fails to consider two processes: the "buy-up" to higher fares if discounts are unavailable, and the "buy-down" to discounted fares if the latter are available. This is all the more important in markets where customers have access to a wealth of information, and are thus able to compare prices for the same products offered by different companies, or prices for similar products offered by a single firm. Customers strategic behaviour must be taken into account, which shifts the focus of the demand model towards the customer.

The optimal pricing problem on a network (NPP) we consider involves a firm acting in a competitive environment. Given price schedules set by its competitors, the leader firm (upper level) strives to maximize the revenues raised from tariffs set on a subset of arcs of a transportation network, taking into account the reaction of the users of the network (lower level) who rationally minimize their individual travel cost. Neglecting congestion and assuming that demand is fixed, users of the network are simply assigned to shortest paths linking their repective departure and arrival nodes. This sequential and non cooperative decision-making process can be adequately represented as a bilevel program (cf Labbé et al. [7]). It has been recently proved to be strongly NP-hard by Roch et al. [8] and Grigoriev et al. [5].

In this presentation, we describe a tabu search algorithm designed to solve large size instances of the NPP problem and give preliminary numerical results. It is important to notice that even if this presentation is devoted to a simplified pricing problem, results are of the main importance since they are basis stone for further developments.

2 Tabu Search algorithm for NPP

Introduced by Glover [4] and Hansen [6], Tabu Search (TS) allows a thorough search of the solution space to circumvent early stoppings of algorithms due to local optima. It has been applied to solve large size of a large variety of problems. To our knowledge only Gendreau *et al.* [3] proposed a tabu search algorithm to solve linear linear bilevel programs.

At any iteration, the aim of TS is to find a new solution by making local movements over the current solution. The next solution is the best among all possible solutions in the neighbourhood of the solution. To avoid cycling, once a solution is visited, the movement from which it was obtained is considered as tabu by mainting a tabu list. To improve the efficiency of the exploration process diversification strategies are applied repeatedly after a given number of iterations.

In what follows we define a commodity as being a set of users with the same origin and destination on the network. A NPP solution is composed of a follower problem solution and a set of tariffs compatible with this solution. More precisely it is defined as a set of tariffs and a set of commodity shortest paths corresponding to those tariffs. We outline hereafter the main features of the tabu search heuristic.

2.1 Neighborhood structure

The tabu search heuristic aims to explore follower problem solutions providing high leader revenue. The neighbours of a follower problem solution are obtained by removing one arc u and inserting one arc v into the shortest path tree associated with a given commodity k. This move is noted (u, v, k).

2.1.1 Neighborhood evaluation

The value of a move is characterized by the optimal revenue associated with a follower solution. It is obtained by determining the optimal tariffs compatible with the fixed commodity paths. More precisely it means that the commodity paths are the solution of the shortest paths problems corresponding to the tariffs. This problem, named inverse optimization program, can be solved by considering its dual which is a generalized multicommodity flow problem. The 'generalized' characteristic comes from a constraint requiring a global null flow on each tariff arc. A path formulation of the iverse optimization dual problem is efficiently solved by applying a Dantzig Wolfe decomposition algorithm [2].

The (u, v, k) move considered is the one generating the best leader revenue among the neighbour non tabu solutions and improving the best current solution. If no such move exists, the solution minimizing the next function is considered:

$$\Pi(u, v) = [t_e(u) - t]^+ + [t_s(u) - t]^+ + 2[t_p(u, v) - t]^+,$$

where $[a]^+ = \max\{0, a\}$, t in the current iteration number, $t_e(u)$ is the iteration index for which u has been inserted in the Tabu list L_e, $t_s(u)$ is the iteration index for which v has been inserted in the Tabu list L_s and $t_p(u, v)$ is the iteration index for which u, v have been inserted in the pivot list L_p. A detailed description of the tabu lists will be given in the next section.

Each (u, v, k) moves requires to update the commodity shortest path trees. More precisely, whenever the selected solution does not improve the revenue associated with best known solution, the commodity tree updates result from simplex pivots. If it is not the case then the tree updates result from the computation of the commodity optimal paths corresponding to the tariffs associated with the improving solution.

2.1.2 Neighborhood reduction

To speed up the proposed tabu search approach, we only consider a commodity subset of k_1 commodities at each iteration. At the intial step we consider the k_1 commodities with the largest demands. Then this set is updated with the $k_1/2$ commoditites leading to the generation of the best solutions at the previous iteration and with $k_1/2$ commodities unfrequently considered.

2.2 Tabu lists

Cycling among solutions is prevented by considering three tabu lists defined as follows.

- L_e: Entering arcs and associated commodity tabu list.
- L_s: Removing arcs and associated commodity tabu list.
- L_p: Pivot and associated commodity tabu list

With each tabu list element is assigned a tabu status for θ iterations where θ is randomly selected in the interval $[\underline{\theta}, \overline{\theta}]$. Note that a solution is named "tabu" if either the associated entering arc or removing arc or pivot is tabu.

2.3 Diversification strategies

The diversification strategy aims to get away as far as possible from a follower solution to extensively explore the solution set. In the diversification phase, we apply TS with a modified objective function defined as follows. Le S be a follower problem solution and S^* the best current follower solution. The objective function $Z_{div}(S, S^*)$ is defined as:

$$Z_{div}(S, S^*) = Z(S) + Z(S^*)D(S, S^*),$$

where $Z(S)$ represents the optimal revenue associated with the solution S and $D(S, S^*)$ is a similarity measure between the solutions S and S^*. More precisely it is defined as the ratio of the number of different paths among the two follower solutions over the total number of commoditities.

3 Numerical Results

The tabu search algorithm has been tested on several families of randomly generated networks. We report in this abstract some results on complete grid networks with 60 nodes and 208 arcs (15% of tariff arcs) that promote interaction between commodities and makes for problems that are combinatorially challenging. The generation random instances is described in Brotcorne *et al.* [1].

Each line corresponds to an average taken over 5 problem instances. Label *Nbcom* represents to the number of commodities. The three labels in the MIP section concerns the exact solution obtained by applying CPLEX on a MIP equivalent formulation of the NNP. Label *NOpt* refers to the number of instances solve to optimality within a time limit of 43200 seconds. Label % is defined as $100(Z_{sup} - Z_{inf})/Z_{inf}$ where Z_{inf} (resp. Z_{sup}) is the lower bound (resp. the upper bound) on the revenue obtained when the resolution is stopped. Label CPU represents the CPU time in seconds. Concerning the tabu search algorithm (section TS), label % is defined as Z/Z_{inf} wher Z is heuristic solution objective value and Z_{inf} the exact one or the best lower bound on the revenue obtained if the optimal solution could not be obtained within the time limit. Label CPU gives the computation time in seconds. The number of iterations of the TS is fixed to 2000 and $k_1 = 25\% \times K$.

Nb	**MIP**			**TS**	
com.	NOpt	%	**CPU**	%	**CPU**
10	5	0.00	30.98	99.79	11.92
20	4	0.91	16689.36	100.00	36.38
30	2	3.37	39566.84	100.09	61.06
40	0	17.53	43492.37	111.33	95.24

Numerical results show that the tabu search heuristic we design to solve the NPP problem provide good quality solutions in small computation time. In less than 2 minutes, TS produces solutions with better or the same quality than the ones obtained with CPLEX in larger computation times. As will be illustrated in the presentation this can be generalized for other instances families. These encouraging results comfort our belief that this methodology can be extended to more general pricing problems of the same type.

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