MODELING ROUTE CHOICE SETS IN TRANSPORTATION NETWORKS: A PRELIMINARY SYNTHESIS.

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1. Introduction

Choice sets of individual travelers play a paramount role in analyzing travel choice behavior. Choice sets are defined as the collection of travel options perceived available by individual travelers in satisfying their travel demand. From a variety of studies (e.g. Swait and Ben-Akiva ,1985, 1987) it is well known that the size and composition of choice sets do matter in cases of choice model estimation and demand prediction. Incorrect choice sets (e.g. because of captivity) can lead to misspecification of choice models and to biases in predicted demand levels (see Ortuzar & Willumsen, 2001, and Williams & Ortuzar, 1982). While this has been demonstrated for relatively simple choice types such as mode choice, we may assume that it holds as well for the more complex case of route choice (see VanDerWaerden et al.(2004), for an example, without however giving explanations for their findings). The critical role of choice sets in choice modelling has given rise to profound research into choice set modelling in the transportation field, although largely confined to mode choice (see e.g. Swait, 2001). We state that these insights gained on choice set modelling and choice set generation cannot simply be transferred to the route choice realm. For a variety of reasons, the specification of route sets for route choice modelling is different and more complex, reason why this topic deserves special attention from researchers and practitioners.

This paper is devoted to a number of topics related to the modelling and generation of route choice sets, specifically for application in large networks including multi-modal networks. The paper above all tries to synthesize existing knowledge on this topic into a single conceptual framework giving ample attention to a theoretical underpinning.
We will first summarize in what respect route choice sets differ from other travel choices implying that some proposed choice set modelling approaches cannot be adopted for routes. Then we will argue that it is necessary and advantageous to distinguish the processes of choice set formation and choice per se on the part of the traveller, but also to explicitly separate the modelling steps of choice set generation and choice modelling from given choice sets instead of implicitly combining these steps into a single one. Before going into more detail of route choice set modelling, we will discuss the various notions of choice sets and the various purposes for which choice sets may be used. We state that these different purposes, that is supply analysis, model estimation, and demand prediction do matter in choice set modelling. We then present a generic conceptual scheme relating the distinct key elements

inherent in each route choice set generation approach. This scheme helps in classifying and characterizing the various approaches proposed. This scheme is then followed by a generic formalization of the route generation modelling procedures. Some indications for their empirical validity will be presented derived from applications to various uni-modal and multi-modal networks.

2. Special aspects of routes in choice analysis and demand prediction

In the following we will elaborate on choice set generation for the establishment of route choice sets in networks for various purposes. In doing this we have to bear in mind a number of specific characteristics of route choice sets, such as among others, the following:

- the population of available routes for a trip (the so-called universal set) in dense networks usually is very large and mostly not known; sometimes hundreds of routes can be identified for a single OD-pair;
- the subset of feasible and attractive routes often also is very large. In addition, this set is complex because of heterogeneous route composition and varying physical overlap among the routes;
- because of these aspects, it is difficult or even impossible for the analyst to enumerate the full set of routes attractive to the travellers and relevant in choice modelling;
- the identification of a route as a genuine travel alternative is not a trivial task because of complex patterns of overlap that are even more severe in public transport networks because of the time dimension of alternative services;
- The loading pattern (congestion, queues, spillback) of the network (for road and public transport networks) has a high impact on route attribute values and consequently on the composition of the set of attractive routes.

With respect to modelling of route choice, the following specific aspects play a role:

- in principle, each trip (each OD-pair) has a different choice set in terms of size and composition, ranging from very small to very large and complex;
- generally, routes cannot be given unambiguous observable labels and can only be described by generic attributes in stead of alternative specific attributes;
- choice behaviour among routes is ambiguous and may consist of various forms of choice process, such as sequential (from decision point to decision point, see e.g. Marzano & Papola, 2004), simultaneous (from origin to destination at once) or strategic (adaptive choice based on prevailing network conditions during the trip);
- the networks used for modelling mostly are simplified versions of the real network so that generated routes cannot exactly be identical to their real counterparts;
- the impact of heavy loads should be taken into account.

Additional specific aspects are apparent in the case of routes in multi-modal networks (see e.g. Hoogendoorn-Lanser 2005, and Fiorenzo-Catalano, 2007), such as:

- even more alternatives (larger and more diversed choice sets) due to additional choice dimensions such as choice from public transport stops, transfer nodes, access modes, egress modes, line haul modes, etc.;
- limited feasibility of multi-modal routes because of modal sequences within the trip.

The absence of a well-defined universal choice set requires the adoption of special generation methods for establishing route choice sets in real networks. In addition, the sheer size of networks and numbers of OD-pairs ask for an efficient model-based computational procedure. For these reasons, procedures developed for choices among modes or brands (for an overview see e.g. Andrews and Srinivasan, 1995) are generally not applicable to route choice.

3. Distinction of choice set formation and choice from a choice set

For sake of elaboration of our subject, we introduce a conceptual framework on the decision-making problem of route choice from the traveler's perspective. This framework (Figure 1) is a synthesis of scientific knowledge on this topic most of which is corroborated by empirical findings (for details see Bovy & Stern, 1990; for a synthesis referring to the field of brand marketing, see Andrews and Srinivasan, 1995).
At the right side, the figure shows a series of route sets that follow from a variety of experimental and mental processes (shown to the left) of the traveler. At the left side properties of the traveler are listed that influence his decision-making.

The network usually offers a large and complex set of route alternatives for a trip of which the traveler has only limited awareness (cognition). In big networks the universal route set cannot be known. This cognition is associated among other matters with the traveler's experiences (see feedback from chosen routes) and his manner of acquiring information about travel opportunities. His awareness set will be influenced by his travel preferences: time sensitive travelers will look for other alternatives than cost sensitive persons. The traveler will not always consider all known alternatives to be genuine travel options since there may be several constraints set by the traveler and his travel demand that preclude using some of the known routes, such as time, cost or comfort limits. Those known alternatives that satisfy his constraints will form the choice set available from the perspective of the traveler. The traveler's choice will depend on particular, not necessarily directly measurable characteristics of the routes, his choice factors. The subjective values of these factors follow from his perceptions of objective route attributes relevant for his trade-off and choice. The traveler's perceptions of relevant alternatives and their attributes are incomplete and inaccurate and again are linked to his travel experiences and travel preferences. Sometimes routes are not considered as distinct alternatives because of high mutual overlap. Not all relevant route characteristics are equally important to the traveler. Based on a factor-importance hierarchy the traveler will first sift through the fairly sizeable set of available alternatives to rule out all those alternatives that prove not to be sufficiently satisfactory or useful when considered by aspect (elimination by aspects). Some travelers preclude the use of motorways or tolled roads, whereas others may disregard routes having a detour of more than a factor two or a number of public transport transfers of three or more. Only after a fairly limited group of attractive feasible alternatives remains (the consideration set), the traveler will make a more in-depth evaluation of options making a trade-off among their counter-balancing characteristics using some composite combination rule.

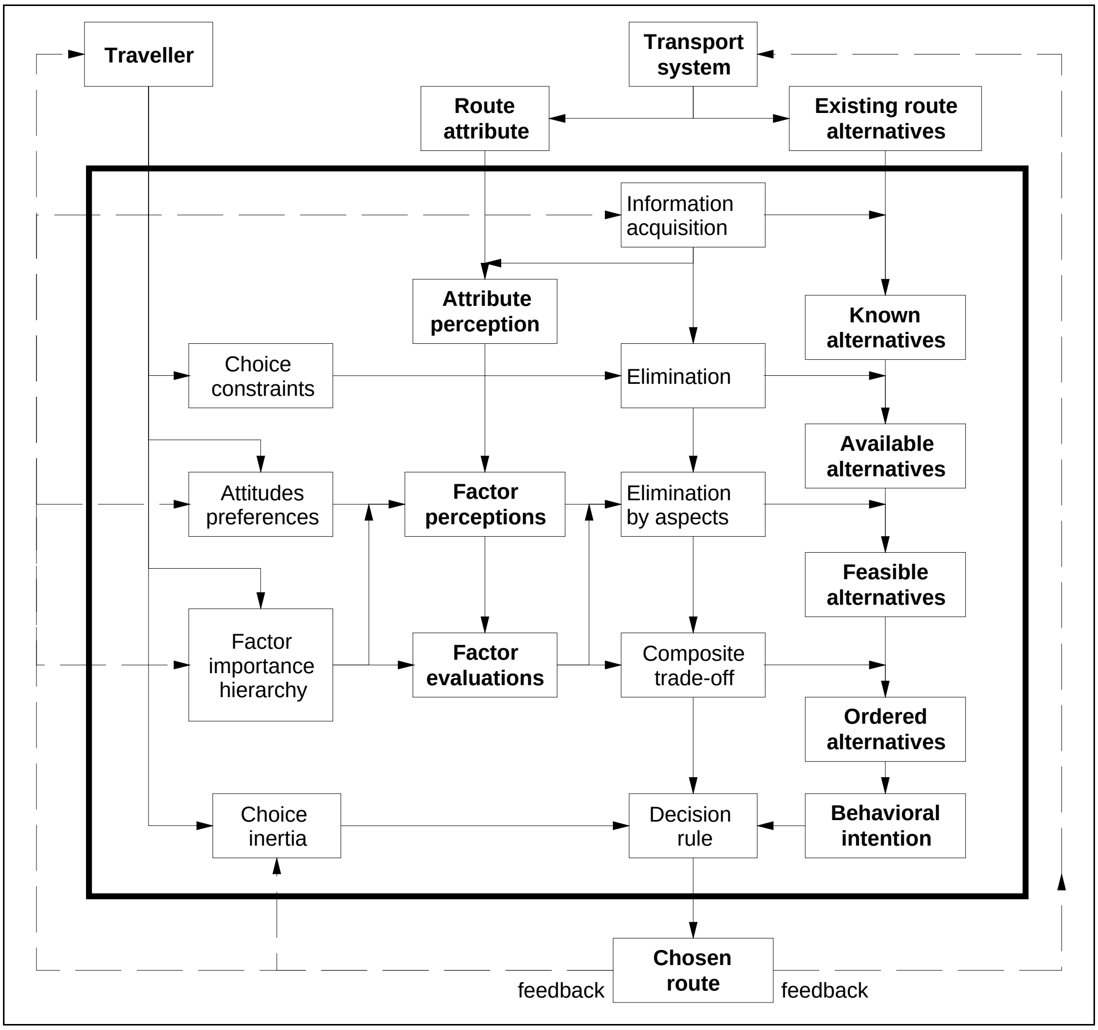


Figure 1: Conceptual framework for choice set formation of an individual traveller (Source: Bovy and Stern, 1990).

The route choice decision-making is characterized by dynamic components, the most important being feedback from usage of chosen routes to nearly all phases in de decision process. In this way, choice set formation is a learning process through dynamic adaptations of cognition and perception of available and feasible options (see e.g. Richardson, 1982). Given the type of processes involved in route choice decision-making, we may expect strong individual differences in behavior due to differences in cognition, perceptions, and preferences of options among individual travelers. Despite these strong idiosyncracies, different individuals may however take the same decisions, that is, choose the same route, though on different grounds.

For our choice set formation subject we may conclude the following:

- choice set formation and choice from considered options are *distinct mental processes* of a traveler that follow different rules;
- whereas choice set formation predominantly is governed by *non-compensatory* decision-making on the basis of constraints and elimination by aspects, choice from

considered options appears to be largely of a compensatory nature (see e.g. Recker and Golob, 1979);
- apart from constraints, choice set formation is also *preference driven*, influenced by the traveler's most important choice factors (see e.g. Horowitz and Louviere, 1995);
- a traveler's choice set formation largely follows an experimental process of *trial-and-error* of route use and information acquisition (see e.g. Richardson, 1982);
- consideration sets may differ strongly between individuals even under the same conditions (see e.g. Basar and Bhat, 2004).

While the discussion above refers to choice behaviour under real conditions, the situation with stated preference experiments is not fundamentally different. Although the analyst will have full control over the presented alternatives (the universal set), the set of alternatives actually considered in the subject's trade-off (his consideration set) may be a subset not directly known to the analyst (see Louviere et al, 2000).

Given these insights, it is imperative to try to establish modeling approaches of travel behavior as closely as possible to these behavioral facts. For the modeling of choice sets this implies the adoption of a distinct separate choice set generation step, preferably in a probabilistic sense because of the unobservability of choice sets actually considered by the travelers. Separate modeling of choice set formation and choice is advantageous both in cases of model estimation and demand prediction because it offers ample opportunities for controlling the required variety in choice set composition as well as for flexible and adequate modeling of each of the two distinct processes (see also Cascetta and Papola, 2001). Explicit choice set generation is especially needed in the route choice context because of the sheer size of networks and because choice sets differ between OD-pairs. In the case of choice sets for choice model estimation, information derived from the observations (such as on constraints and preferences) may help in establishing traveler-specific choice sets.
Explicit distinction between choice set generation and choice in a two-stage modeling process is also advantageous from a policy impact point of view because the analyst can attribute the impacts of a network intervention (adding/removing links, changing link attributes) to an enlarging of choice sets, to changed attractiveness of particular alternatives, or to both (see e.g. Basar and Bhat, 2004).

Specifically for the traffic assignment part of travel demand prediction in big networks, a separate choice set generation step prior to the assignment calculations offers a number of theoretical and computational advantages (see Bliemer & Taale, 2006) although special attention is required to adequately handle the impact of heavy congestion. In contrast to the most usual implicit route generation during iterative network assignment approaches (see e.g. Nielsen et al 2002) it can be shown that specification of route sets in advance of the route/departure time choice predictions and flow calculations has a number of advantages:
- it can deal more easily and adequately with typical route properties such as non-linearities and mutual overlap;
- it offers greater flexibility and realism in choice model types to be adopted;
- it saves computation time, especially in the case of dynamic or multiclass assignment tasks because route search needs to be done only once while equilibration of flows converges much quicker because flow distribution over multiple paths happens from the very beginning.

(It should be noted that the implicit route set generation approaches based on the concept of reasonable routes (see e.g. Marzano & Papola, 2004) are heuristics that fail to generate the full set of attractive routes).
These ideas about route choice set generation will be elaborated further in Section 6 after having introduced specific notions of choice sets and a discussion on purposes of choice set generation.

4. Notions of choice sets from traveler's and analyst's perspectives

With the general notion of *choice set* we define the collection of travel options available for making a trip by an individual or set of individuals. In this respect we need to distinguish between the traveler's perspective and the modeler's perspective. Whereas we may assume that the traveller has a deterministic view on his choice sets, the analyst in contrast usually does not know the choice sets actually considered by the traveller and needs to resort on probabilistic or fuzzy statements about choice sets of travelers.
We use the term *universal set* for the collection of all possible routes between an origin and a destination not limited by any constraints on the part of the traveller(s). Networks usually are characterized by very big universal route sets, mostly not knowable or enumerable by traveller or analyst.

We define as the *subjective consideration set* the subset of alternatives available, feasible, and known to an individual traveler for a particular trip given his personal conditions. It is a subset of his awareness set. Such sets are usually very small and idiosyncratic given the particular conditions of individuals and specific trips, such as vehicle availability, time constraints, maximum acceptable detour, and the like. Only in case of sufficient information about the traveler's specific conditions, the modeler can make probabilistic predictions of the consideration sets at hand, called *generated* consideration sets, needed for choice modeling. This may hold for the case of estimation of choice functions based on individual observations of travel behavior. Usually, the generated consideration set will be larger than its subjective counterpart. Horowitz and Louviere (1995) offer an extensive discussion on the various ways of observing and operationalizing consideration sets, while Hoogendoorn-Lanser (2005) discusses this topic specifically for the case of multi-modal route consideration sets.

In the case of travel demand prediction with given choice models, usually no information is available on an individual level but rather on flows between areas. At most, some information is known at the level of the population of travelers, such as percentages of travelers with particular forms of vehicle availability or levels of income. Therefore, in the prediction case, the modeler tries to establish a *collective* choice set for a particular interzonal travel demand that fits to that collection of travelers by trying to embrace the population of individual consideration sets into a single *collective* consideration set. Since preferences, information levels and other conditions usually are very different among individuals making a similar trip, the collective consideration set will be much larger and more heterogeneous than any individual consideration set.

Since universal route sets usually cannot be known, the analyst needs to specify a base route set from which the generation of consideration sets may be derived. Such a base set mostly is generated from a simplified modelled version of the real network. In the following we call such a set the *master set*. Using a set of criteria this master set is well-defined (from the analyst's perspective) and its size and composition can be known, although it is not yet a true choice set because it may entail non-feasible alternatives (see Section 6).
For a more elaborate discussion on choice set notions from the traveller's and modeller's perspectives, see Hoogendoorn-Lanser (2005).
In the following we will use the term 'consideration set' specifically for the route sets used in the choice modelling stage, whereas the generic term 'choice set' is used otherwise.

5. Purposes of route choice sets

Route choice sets may be generated mainly for the following three application purposes:

- Supply analysis of travel options in networks where the planner or researcher is interested to know the availability of travel alternatives, their number, their characteristics, their variety, their composition etc.;
- Demand model estimation (e.g. estimating behavioural parameters of utility functions of choice models at individual level);
- Prediction of choice probabilities or shares in a demand analysis for determining route flow and link flow levels in networks using route choice models with known parameters derived from estimation, mostly at zonal level.

These different applications of route choice sets appear to pose different requirements on size and composition of the choice sets to be generated.

Whereas choice sets (observed or generated) need not necessarily be exhaustive for estimation purposes, prediction choice sets must include at least all attractive routes. For estimation purposes, even if not all relevant alternatives are included and small well-sampled choice sets are considered in the estimation model, it may nevertheless provide satisfactory results (Ben-Akiva & Lerman, 1985), see Van der Waerden et al (2004) for a demonstration in a route context. On the other hand, in the context of a prediction application, the analyst is interested to achieve satisfactory predictions of route and link flows, especially for those routes and links that have special policy relevance. Such a prediction involves calculating the choice probabilities of all non-zero OD-trips, maybe separately for user groups or trip purposes, and then summing up the number of trips that will use each of the potentially feasible routes, and derived from this, through a network assignment, the use of links. This would require the specification of choice sets in which each route that may attract trips is included. In the case of predicting route flows, the requirements on the quality of choice sets are very strict since in order to have correct route flows, predicted choice sets should include all relevant routes. Inclusion of some unattractive routes in the choice set is not expected to distort the demand predictions (although there may be an effect via inter-route correlation due to overlap), nor will these have serious influence on computational efficiency.
Consequently, a generated prediction choice set should likely consist of all relevant routes with high probability of being chosen; inclusion of some unattractive routes is acceptable.

For a variety of reasons given in Section 3, explicit generation of route choice sets is advantageous both for estimation as well as prediction purposes. We stress that for prediction applications, explicit generation of route choice sets *prior* to route and link flow calculation instead of during the iterative flow calculations is strongly favoured if loading patterns in the network are adequately taken into account.

6. Modeling of route choice sets

In this section we will introduce a generic scheme (Figure 2) for sake of comprehensively characterizing and classifying various approaches to choice set modeling, in particular with respect to routes. This scheme is an application-oriented operationalization of the conceptual scheme (Figure 1) discussed in Section 3 while at the same time synthesizing the insights and findings reported in the general consideration set literature (see e.g. Andrews and Srinivasan (1995), Ben-Akiva and Boccara (1995) or Basar and Bhat (2004)). The scheme starts from our statement that especially in the case of routes in networks, a separate choice set generation step is highly preferable in contrast to proposals for integrating the generation and choice model processes into one single model (see e.g. Swait, 2001).

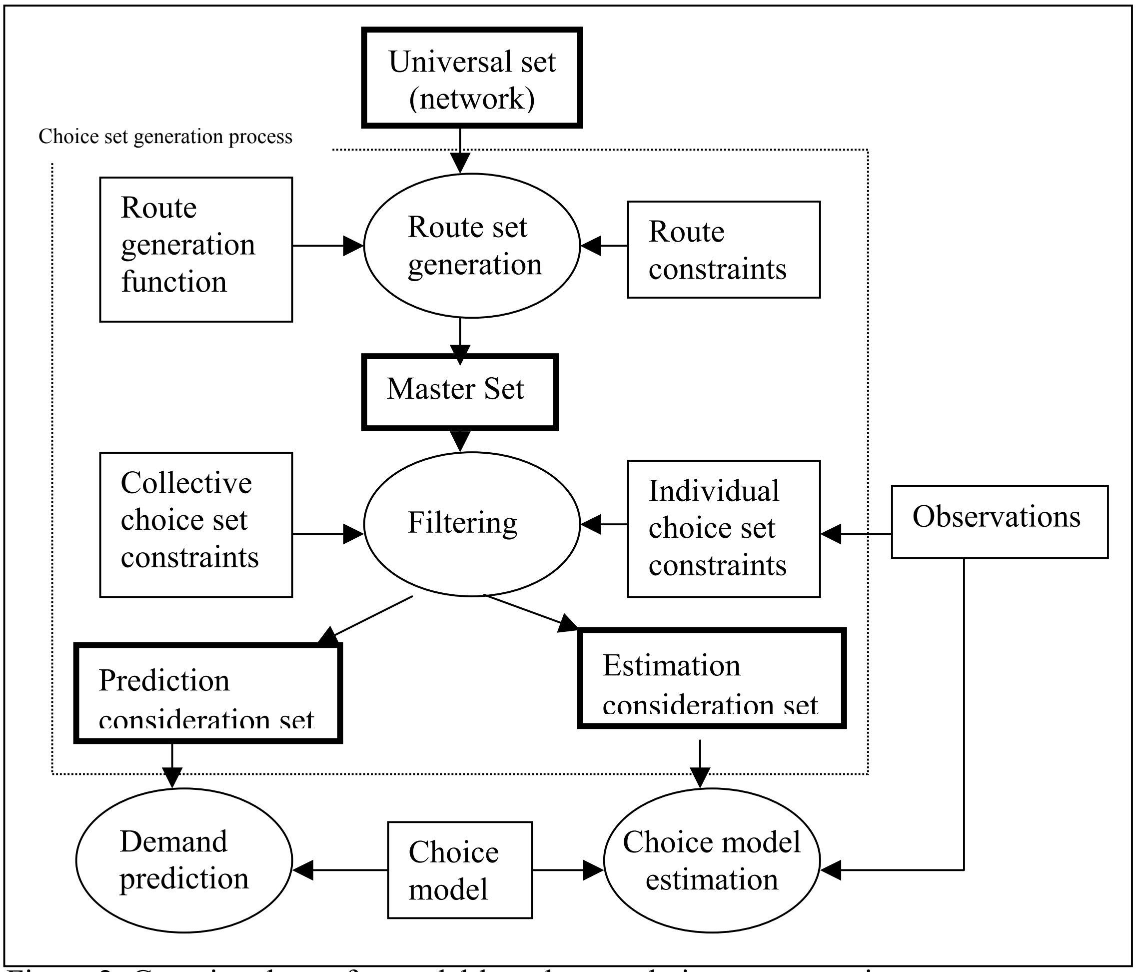


Figure 2: Generic scheme for model-based route choice set generation

According to our theoretical framework we conceive of the process of model-based choice set generation to consist of two basic steps: first, the *generation* of a *master set* based on a universal set of routes the size and composition of which in case of usual networks is not known to the modeler. Subsequently a second step, called *filtering,* is performed where the master set is narrowed down deterministically or probabilistically to a *consideration set* by applying a set of constraints to the master set although for some non-route travel choices (with only a small universal set) these two steps may be collapsed into a single one.

In order to be realistic, generated choice sets need to fulfill a number of *requirements* of a logical and behavioral kind, depending on the purpose of the sets. In estimation applications, route choice sets need to reflect the cognitive and preferential properties of the individual traveler. These requirements include among other matters: loopless routes, directional efficiency, time feasibility, maximum detours in distance and travel time relative to straight line and shortest path, hierarchical set-up of routes according to functional class of its constituting links, sufficient distinction of a route relative to others, etc. At this individual level, specific properties of the traveler known from observations might be taken into account, such as vehicle availability, familiarity with the network, and the like. In prediction applications, choice sets also should reflect the variability in cognition, preferences, spatial conditions and other properties among travelers between the same origin and destination. The requirements are typically of a non-compensatory nature.

For an in-depth elaboration of choice set requirements for different modeling applications, see Hoogendoorn-Lanser (2005) and Fiorenzo-Catalano (2007).
In the choice set generation approaches, the choice set requirements are translated into corresponding search functions and constraints (see below).

Given the model-version of the network, the generation step generates candidate routes for the master set by some algorithm (also called generation function). In case of routes, the master set is different for each OD-pair. The generation function may partly express the role of preferences in the choice set formation process in that variables expressing preference (being part of the choice function) also are part of the generation function (for example time and cost). Most known algorithms use shortest path search as their basic generative operation. Usually at the generation stage a set of constraints is applied that each candidate route should satisfy. These so-called *route constraints* are of a general nature by removing on a non-compensatory basis routes that do not fulfill logical requirements and generally valid behavioral constraints. Route constraints mainly screen for the feasibility of single routes. In the recently developed constrained enumeration algorithms for master set generation based on the branch-and bound technique (see Section 7), built-in constraints reflect the preferential and cognitive requirements.
The *filtering constraints* on the other hand are much less generally valid and depend on the specific case (trip type, traveler type, network type, etc) and specific purpose at hand (estimation or prediction). In contrast to route constraints, the filtering constraints predominantly screen for the composition of the choice set (in a deterministic or probabilistic manner) by looking at relationships among the routes in the master set. Routes may be removed because of several reasons:

- high mutual overlap: routes may not be considered as distinct alternatives because of high spatial or temporal overlap;

- hierarchical structure (non-feasible sequence of functional class of constituting links);
- non-availability to the observed individual at hand,
- failing to meet the minimum consideration utility level (see Andrews and Srinivasan, 1995, and Basar and Bhat, 2004); etc.

A filtering step always is needed since some quality checks (e.g. spatial variability) are only possible after the master set generation step has been completed.
For a comprehensive discussion and application of such constraints with deterministic implementations, see Hoogendoorn-Lanser (2005) and Fiorenzo-Catalano (2007). In the pedestrian route application of Van der Waerden et al (2004), for each observation a master set of 50 routes was generated from which consideration sets of 10 routes were selected using a set of rules.

The master set is considered to be of a deterministic nature although it might be produced by a probabilistic or stochastic approach. On the other hand, the consideration set is considered to be of a probabilistic nature because it is a generally unobservable set (latent set). To that end, apart from deterministic constraints, one or more probabilistic filtering constraints may be applied. Probabilistic filtering approaches determine the probability that a route is part of the traveler's consideration set. An example of such a probabilistic constraint is the consideration utility threshold that should be exceeded by the alternative's consideration utility derived by some function of the alternative's attributes (see Andrews and Srinivasan, 1995, and Basar and Bhat, 2004). From these route consideration probabilities one can derive the probability that a particular route subset is the consideration set.
Most extant choice set generation methods (including the implicit ones) however approximate the probabilistic filtering with deterministic constraints (e.g. valid for 90% of the trips) giving rise to deterministic consideration sets (see e.g. Van der Waerden, 2004). These may be characterized as rule-based elimination-by-aspects filtering procedures.

In principle, this two-stage generation process with two sets of constraints is applicable to the estimation (right side of Figure 2) as well as the prediction purposes (left side). The difference is in the adopted constraints, especially in the filtering step. In choice model estimation, observations of individuals are available (chosen alternative(s), vehicle ownership, network familiarity, and the like) from which individual-specific (random) constraints can be established for the filtering step. In a prediction application, the constraints have to resort on more general statistical data of the population of travelers, such as for example distributions of minimum and maximum walking and cycling distances, of maximum number of transfers, of willingness to use motorways or tolled roads, and the like.

Adoption of the random constraints approach so far has been limited to very simple choice situations with a master set of only a few alternatives such as mode choice (see e.g. Swait and Ben-Akiva (1987), Ben-Akiva and Boccara (1995), Louviere et al (2000) and Basar and Bhat (2004)). It seems impossible to adopt a full probabilistic approach (according to Manski's (1977) unconditional probability formulation) to route set generation because of the sheer size of master sets in networks, even after strong selection. If a master set for a particular OD pair consists of about 10 routes (which is a very small size indeed), calculating the route probabilities seems feasible. However, since 10 routes imply $2^N-1 = 1023$ potential

consideration sets, determining these set probabilities and using these for example in a choice model estimation exercise seems beyond current possibilities.
An intermediate approach, the so-called Implicit Availability/Perception (IAP) model, has been proposed where the probability of choice set membership of an alternative (of a deterministic choice set) enters the utility function of the choice model (see Cascetta and Papola, 2001, Cascetta et al, 2002) where a low membership probability of an alternative naturally leads to a decrease of its choice probability.

The output of the choice set generation process thus may be:
- a deterministic consideration route set per OD-pair;
- a set of choice sets with corresponding consideration probabilities, per OD-pair;
- a set of routes with corresponding consideration set membership probabilities,

either of which is then input into the subsequent choice modeling step.

Whereas the consideration set generation model and choice model are specified separately, the parameters of both models maybe estimated jointly in a single estimation exercise (see examples of Andrews and Srinivasan (1995), Ben-Akiva and Boccara (1995) and others). It is questionable whether such a complex estimation endeavour is desirable and feasible in the case of route choice.
For a formalization of this 2-stage route set generation approach, see Section 8.

7. Overview of choice set generation approaches

We will now use the proposed two-stage generation scheme of Section 6 to give an overview of known generation procedures for route sets and to identify their specific properties.

We may distinguish three classes of explicit generation procedures for route sets (Table 1):
- probabilistic methods (the IAP-model);
- constrained enumeration methods.
- path search based methods;

The IAP-model is based on a given master set determined by some deterministic method (e.g. the labeling method, see below). In the filtering step, for each route in the master set a consideration set membership probability is calculated based on observed individual data. The membership probability model is mainly based on perception variables of the traveler. The consideration set thus equals the master set with added membership probabilities that enter the route’s utility function in the choice modeling step. Cascetta and Papola (1998) and Bekhor et al (2001) offer applications of this IAP-approach to route choice in urban networks demonstrating its feasibility.
All other known route set generation procedures are deterministic, most of which are restricted to the generation of master sets.

The *constrained enumeration procedures* generate an *exhaustive* master set given a set of constraints. This procedure constructs a connection tree between origin and destination of a trip by processing sequences of links according to a branching rule. The generation function in this case is the building of the tree. The extension of the tree with additional links (until the destination has been reached) is conditional on a number of built-in constraints that reflect cognitive, perceptual, and behavioral requirements (directionality, maximum overlap, maximum detour in distance and time, etc). The types of adopted constraints imply that the choice set generation predominantly is preference driven. Friedrich et al (2001) developed an application to public transport networks (Germany), Prato and Bekhor (2006) to road networks (Turin), while Hoogendoorn-Lanser (2005) established an extensive implementation for multi-modal networks (Randstad Region Netherlands), see also Hoogendoorn-Lanser et al (2006).

Table 1: Classification of route choice set generation approaches

Choice set Generation method	*Univer-sal set*	*Master set generating function*	*Generating constraints*	*Filtering constraints in parameter estimations*	*Filtering constraints in demand prediction*
Probabilistic IAP model	network	Shortest path criteria (selective)		On perception and availability (deterministic)	Preference based
Constrained enumeration methods	network	Tree building (exhaustive)	Preference and cognition based		Preference based
Deterministic shortest paths methods	network	Shortest path criteria (selective)		Preference and cognition based	Preference based
Stochastic shortest paths methods	network	Stochastic shortest path criteria (selective)		Preference and cognition based	Preference based

Because master sets generated this way may be very large with insufficient distinction between routes, a filtering step is recommended (Hoogendoorn-Lanser, 2005) with more strict and additional constraints aiming at sufficient hierarchical quality, spatial and functional variety, and the like. This is the more needed for establishing consideration sets to be used for choice model estimation purposes.

The largest group of generation methods is based on *repeated shortest path search* in the network where shortest paths are successively generated by changing one or more of the input variables such as the search criterion, link attributes, and constraints (separately or combined). The generation function in this case is the path search criterion, which is mostly based on the more important choice factors such as travel time, distance and the like. Because of this principle the methods are strongly preference driven. These methods generate a *selective* set meaning that not all potentially relevant routes will be generated. Essential characteristic of these methods is the optimization principle inherent in the shortest path search. It implies that these methods predominantly generate the more attractive routes. Table 2 summarizes a number of well-known methods with their specific characteristics. Columns 2 to 4 indicate the sources of variation in successive path search, Columns 5 and 6 the type of computational result, while Column7 indicates whether the method is applicable to multiple OD pairs simultaneously or only to single OD-pairs successively.

Table 2: Repeated shortest path search based route generation methods.

Route set generation approach	*Change network*	*Change search criterion*	*Apply constraints*	*Procedure*	*Result*	*OD-pair applic.*
1	2	3	4	5	6	7
k-shortest paths	X			Exact	Det.	Single
Constrained k-shortest paths	X		X	Exact	Det.	Single
Link elimination	X			Heuristic	Det.	Single
Link penalty	X			Heuristic	Det.	Single
k-dissimilar paths	X		X	Heuristic	Det.	Single
Gateway method			X	Heuristic	Det.	Single
Essentially least-cost paths			X	Heuristic	Det.	Single
Labeled paths		X		Heuristic	Det.	Multiple
Monte Carlo (MC) shortest paths	X			Heuristic	Stoch.	Multiple
MC labeling combination	X	X		Heuristic	Stoch.	Multiple
Doubly stochastic approach	X	X		Heuristic	Stoch.	Multiple

The listed methods essentially are route set generation methods since a specific recognition of the requirements of choice sets (let alone consideration sets) mostly is absent. To that end, an added filtering step would be required. (The implicit route set generation heuristics such as based on the reasonable route concept (see e.g. Marzano & Papola,2004) are omitted from this overview because they do not identify single routes and fail to consider all attractive alternatives).
For an in-depth description (with references) and assessment vis-a-vis suitability for choice set generation in uni-modal as well as multi-modal networks, see Fiorenzo-Catalano (2007).

8. Formalization of route choice set generation

The qualitative descriptions given in Sections 6 and 7 will now be formalized in a generic modeling framework applicable to all methods known from the literature. The expressions refer to a single choice situation of a traveler for a trip from an origin to a destination or to an OD pair, indicated by index i (sometimes omitted below for clarity).
We use the following notation:
i : trip index (OD-pair);

U_i : universal set of routes, the size and composition of which usually is unknown but is implicitly given by the network description of nodes and links;
M_i : non-empty master set of |M| unique routes, to be determined by some method, $M_i \subseteq U_i$;
MS: power set of M;
C_i : non-empty consideration set of |C| unique routes, to be determined by some method, $C_i \subseteq M_i$;
r : route index in universal set;
V_r : attractiveness of route r according to a single-valued deterministic or stochastic function (the value of which results from a route generation process);
K : set of |K| functional master set constraints expressing behavioral criteria of a logical, cognitive, availability and preference kind;
L : set of |L| computational master set constraints controlling the termination of the generation process according to some criteria (e.g. # of generated routes, # of searches, etc);
$\mathbf{X}$: vector of route attributes;
$\mathbf{x}$: vector of link attributes;
$\boldsymbol{\beta}$: vector of attractiveness parameters;
δ_r : master set membership indicator;
Z_i : set of $|Z_i|$ functional consideration set constraints expressing behavioral criteria of a cognitive, availability and preference kind;
u_{zr} : desirability value of constraining condition z_i with respect to route r for individual i;
p_{zr} : probability that route r satisfies constraint z_i of set Z_i ;
p_r : consideration set inclusion probability of route r;
D : a non-empty subset of M being element of the power set MS of M;
P_r : choice probability of route r.

The generation of the master set is a problem of maximizing the collective attractiveness value of master set M given the prevailing constraints, thus

$$P_M = \arg\max_M \sum_{r \in U} \delta_r V_r \quad \delta_r = 1 \text{ if } r \in M, \text{ else } 0. \qquad (1)$$

s.t. K
s.t. L

In the case of constrained enumeration methods (see Section 7), computational constraints L maybe absent and the route attractiveness of feasible routes are assumed all equal ($V_r = V \ \forall r$). In the case of path search methods (see Section 7), the selected feasible route r has the highest V_r value given the adopted deterministic or stochastic search criterion.
The search criterion 'V' generally is a function of route attributes X_r and traveler preferences β, where most of the route attributes generally follow from the constituent link attributes x.

$$V_r = f(\mathbf{X}_r;\boldsymbol{\beta}) \qquad (2)$$

Repeated path search is performed by varying systematically or stochastically the values for X or β or both. With stochastic methods, the value V_r and therefore also δ_r and M are a single realization from a distribution. Some of the path search methods (such as labeling and Monte Carlo methods) use a goal-oriented search function (2) where the attributes and attractiveness parameters are related to the traveler's behavior (awareness of routes, preferences for attributes, etc). The other path search methods (e.g. k-shortest paths, link penalty, etc) are heuristics that try to approximate the solution to problem (1).

Generating the consideration set C_i is defined as attaching a consideration probability p_r to each route in the master set. This is motivated by the unobservability of the true individual choice set. We hypothesize (following among others Ben-Akiva and Boccara (1995), Morikawa (1995), and Basar and Bhat (2001)) that the consideration set may result from applying a set Z_i of constraints.
For each constraint z_i a latent variable u_{zr} is assumed representing the individual's desirability (a weighted combination of route properties affecting the restriction) of the constraining condition. The z-th constraint is represented by an unknown threshold κ_z to be estimated from observations. The difference between desirability and threshold thus is a random variable. The probability that the constraint is fulfilled equals the probability that the desirability exceeds the threshold:

$$p_{zr} = \Pr ob[u_{zr} \geq \kappa_z] = \Pr ob[E(u_{zr}) - \kappa_z \geq \varepsilon_{zr}] \tag{3}$$

$E(u_{zr})$ is the systematic part of the random desirability function expressed for example by a weighted combination of route properties affecting the restriction (e.g. detours in time and distance).
Assuming the random term ε_{zr} follows a Gumbel distribution, p_{zr} can be calculated with a binomial logit model:

$$p_{zr} = \frac{1}{1 + \exp[\kappa_{zr} - E(u_{zr})]} \tag{4}$$

The probability that alternative r is included in the consideration set equals the probability that route r satisfies all constraints. It then holds (assuming mutual independence of the constraints of route r):

$$p_r = \prod_{z \in Z} p_{zr} \tag{5}$$

where p_{zr} is the probability that route r satisfies constraint z.
This formulation of constraints reflects the mix of compensatory (desirability functions u_{rz}) and non-compensatory (the constraints z) processes in the individual's choice set formation. In the case of Cascetta's IAP model (Cascetta et al, 2001,2002), there is only one multi-variate compensatory constraint z applicable to all routes r of M.

Given these individual route inclusion probabilities one can calculate the probability for each unique non-empty subset D of the master set of being the consideration set. Assuming the constraint sets of different routes are mutually independent, the probability that route subset D is the consideration set of i is given by (see e.g. Ben-Akiva and Boccara, 1995):

$$P(D \mid M) = \frac{\prod_{r \in D} p_r \prod_{r \notin D} (1 - p_r)}{1 - \prod_{r \in M} (1 - p_r)} \tag{6}$$

Knowing the probabilities of each possible subset D allows calculating the unconditional choice probabilities of the routes in the master set using the following expression from Manski (1977):

$$P_r = \sum_{D \in MS} (p_r \mid D).p(D \mid M) \tag{7}$$

where MS is the power set (set of all subsets) of M.

The general probabilistic consideration model formulated above comprises various special cases. If all consideration probabilities are chosen to be 1, the consideration set is identical to the master set (a still often used approach). Some methods only generate deterministically consideration probabilities of zero or one such as the so-called captivity models. Some methods (such as the IAP-model of Cascetta and Papola, 2001) do not derive consideration subset probabilities P(D|M) but instead use the route consideration probabilities p_r as variables in the utility function of the choice model (implying a deterministic consideration set identical to the master set). Given the size of master sets in realistic route choice contexts, it is questionable whether establishment of consideration set probabilities is feasible.

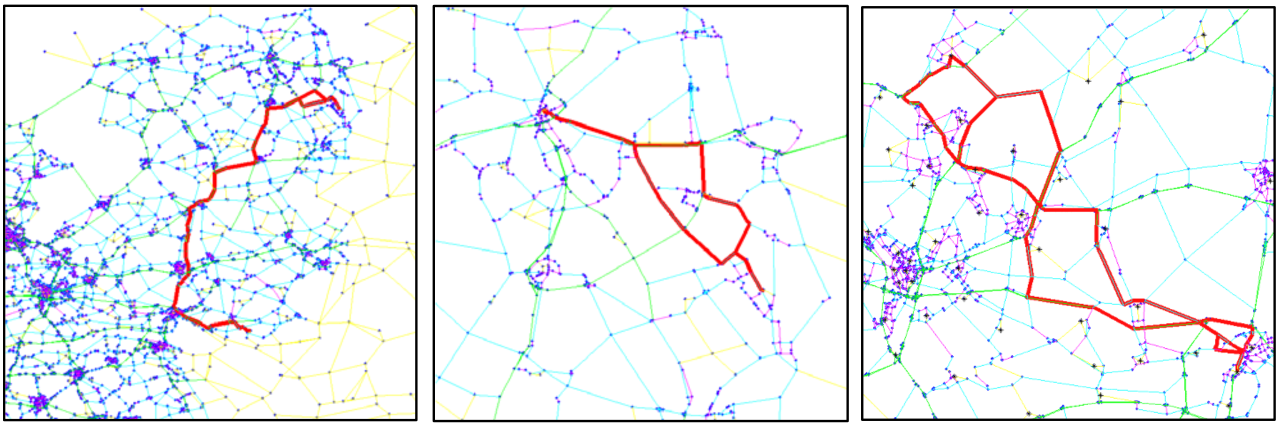
9. Empirical validity of route choice set generation

A general problem in establishing the empirical validity is that mostly a simplified model-version of the real network is used to generate the routes. The validity tests therefore in fact refer to the modeled network instead of to the real network. It has been shown that the quality of the mapping of generated to real routes might be poor especially near origin and destination of the trip, but might be acceptable in between (Bovy and Jansen, 1983). Reflecting that choice set generation modeling in the route choice context is still in its infancy, only few studies have paid attention so far to the validity of choice set generation of route sets. Two studies are known to have collected observations of consideration sets (so-called unaided evoked consideration sets) used for a validity analysis (see Hoogendoorn-Lanser, 2005, and Prato and Bekhor, 2006).
Several criteria may be used to assess the empirical validity of generated choice sets and of the methods by which they are generated.

One may look at their *face validity* or plausibility if dedicated observations are not available: does the generated pattern of routes conform with expectations? Is there enough variability? Is the choice set size sufficient? Etc. See for example the corresponding report in Bliemer and Taale (2006) on an application of the stochastic path search approach to the national Dutch road network (see Figure 3), and on an application to the waterway network reported in Fiorenzo-Catalano (2007). In both applications, plausible route sets with sufficient choice set sizes and spatial variability resulted as input to subsequent traffic assignment computations.

If observations are available (mostly only the chosen route) one may look at the following validity indicators:

- prediction success rate of observed routes (observed routes are reproduced in the generated choice set);
- coverage level of observed routes by the set of generated routes (observed routes are fully or partly covered by generated routes);
- estimation quality of choice models estimated with the generated choice sets.



(a) (b) (c)

Figure 3: Example routes generated for different OD pairs (Dutch national road network): (a) shows an OD pair with only two routes having a large mutual overlap. So, after filtering, only one route entered the choice set; (b) shows an OD pair with two routes that however are clearly different and having a low overlap, hence both routes entered the choice set. The OD pair in (c) has many different partly overlapping routes in its choice set. (Source: Bliemer and Taale, 2006).

Ramming (2002) has probably been the first to establish an extensive comparative study of the quality of various choice set generation procedures (all of the repeated path search class) vis-a-vis observed chosen routes in the road network of Boston by calculating prediction success rates and coverage levels. Prediction success rates range from about 34 % for single path methods to about 60% for multiple path methods. Only the combination of different methods appeared to achieve higher levels (to a maximum of 84%).
In a similar comparative evaluation study using observed chosen routes and reported alternatives in the Turin road network, Prato & Bekhor (2006) determined generated choice set sizes and prediction success rates. Table 3 summarizes their outcomes.

Table 3: Comparison of choice set generation quality (based on data from from Prato and Bekhor, 2006)

Generation algorithm	*Generated unique routes per OD-pair*	*Prediction success rate of chosen route*	*Choice model estimation quality*
Single shortest path (length)	1	53.5	
Single shortest path (free flow time)	1	49.4	
Single shortest path (travel time)	1	43.3	
Single shortest path (delay time)	1	44.5	
Link elimination	5.3	87.2	
Link penalty	6.4	81.3	
Single simulation (small travel time variance)	6.0	75.5	
Single simulation (large travel time variance)	18.2	88.1	
Constrained enumeration	11.2	97.9	best of all

The prediction success rates clearly show that the constrained enumeration approach (not included in Ramming's evaluation) performs best and gives excellent reproduction of chosen routes (98 % of 236 cases were exactly reproduced) and observed alternative routes, even with less generated routes than e.g. the single simulation method. Looking at the choice model estimation quality, using generated choice sets from all (8) shortest path methods combined and from the constrained enumeration approach respectively, various types of route choice models were estimated. The estimation quality (expressed in likelihood values) achieved with the constrained enumeration choice sets appeared to be much better than with the combined methods sets (for details see Prato and Bekhor, 2006), again even with smaller generated choice sets.

Validation results also are available for choice set generation in multi-modal networks. Hoogendoorn-Lanser (2005) developed the constrained enumeration method specifically for multi-modal networks (mixed public and private mode networks). She used reported consideration sets and chosen routes as observations from the Rotterdam-Dordrecht region in the Randstad, The Netherlands. Using constraints based on external data, 91.9 % of chosen routes and 84.5 % of reported consideration sets agreed with the constraints, which represents the prediction success rate. These outcomes are comparable to the ones of Prato and Bekhor for an urban road network (see above). Given the complexity of multi-modal routes this is a very satisfying outcome.
Finally, let us give a report of validation outcomes from a doubly stochastic path search algorithm. Such an algorithm has a stochastic generalized cost function as search criterion (see Formula 2) in which both the link attributes X and the attribute parameters β are randomized (for details see Fiorenzo-Catalano (2007) and Bovy and Fiorenzo-Catalano (2006)). The validation study used a small observation set (35 trips with chosen routes and reported consideration sets) from the same multi-modal data from the Rotterdam-Dordrecht region in the Randstad, The Netherlands. In a number of simulations the prediction success rates (predicting the correct sequence of legs in the chosen multi-modal trip, which means among others correct entry, exit and transfer stations, correct train type and correct sequence of public and private modes) was 89% or higher.

In summary, from the various validation studies it appears that the simple repeated shortest path generation methods (K-shortest paths, labeling, link elimination, link penalty, etc) perform unsatisfactory (for reasons see Fiorenzo-Catalano, 2007). The best performance is achieved by the constrained enumeration approaches. Promising outcomes appear to be given by the doubly stochastic simulation approach, probably because it reflects well the choice set formation process and the required heterogeneity of choice sets.

10. Conclusions and recommendations

Choice sets play a critical role in choice model estimation and demand prediction significantly influencing the validity of parameter estimates and predicted demand levels. Profound research into choice set modeling makes progress in tackling choice set

misspecification problems and in identifying the variables that determine the individual's choice set formation. In the case of route choice, however, advanced methods such as random constraints choice set modeling are not (yet) applicable because of the sheer size of route sets. The analysis of the route choice set specification problem given in this paper can be summarized as follows.
Routes form a specific dimension of travel choice because of unknown size and composition of the universal set, reason why choice set generation approaches established in other fields (e.g. mode and brand choice) cannot be adopted.
On behavioral and practical grounds, a clear distinction and separation need to be made between the modeling of route choice sets (mainly a non-compensatory mental process) and the choice from choice sets (mainly a compensatory process). An advantage of this is that different and more suitable modeling approaches can be used for each part. Prediction of choice sets prior to the prediction of travel demand in networks offers significant advantages from a theoretical and computational point of view although it requires special attention in highly congested networks. From a policy point of view, the recognition of consideration effects can help determine the relative effects of policy relevant variables on consideration and on choice.

In specifying choice sets, different approaches are required depending on the application purpose, be it for supply analysis, choice model estimation, or demand prediction.
Based on theoretical insights on traveler's choice set formation, the paper offered a conceptual framework for modeling of route consideration sets. A two-stage process is proposed consisting of a generation step in which a master set is established from the unknown universal set, followed by a filtering step in which a set of constraints is adopted (possibly apart from a consideration utility function) to narrow down the master set to the desired consideration set.
The framework allows consistent comparison of extant route set generation methods.
Our summary of the few studies into the empirical validity of route set generation models shows promising validation outcomes, especially with the recently developed constrained enumeration approaches.

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