Vehicle routing problem with hard time windows and stochastic service time

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1   Introduction

In this paper we consider the Vehicle Routing Problem with Hard Time Windows and Stochastic Service Times (VRPHTWSST). We can describe the problem as follows. Consider a directed graph $G = (V, A)$ where $V = \{0, 1, \ldots, n\}$ is the node set, and $A = \{(i, j) \mid i, j \in N\}$ is the arc set. Node 0 represents a depot where a fleet of homogeneous vehicles is initially located and $V_c = \{1, \ldots, n\}$ is the customer set. A time window $[a_i, b_i]$ and a stochastic service time are associated with each customer $i \in V_c$. Service time probability distributions are supposed to be known and mutually independent. A non-negative travel cost c_{ij} and travel time t_{ij} are associated with each arc (i, j) in A. Furthermore, a global "reliability" threshold $0 < \alpha < 1$ is given.

The VRPHTWSST consists of finding a set of vehicle routes such that: (i) Routes start and end in node 0; (ii) All customers are served; (iii) Service at customers starts within the given time window. Vehicles are however allowed to arrive before the beginning of a time window. In this case vehicles must postpone the beginning of the service until the customer's time window opens. In no case vehicles are allowed to arrive after the end of the time window. (iv) The global probability that the route plan is feasible with respect to customers' time windows once the customers' service time becomes known, is higher than the reliability threshold; (v) The travel distance is minimized. For convenience, we use

the expression "success probability" of a route to indicate the probability that the route is feasible with respect to customers' time windows once customers' service times become known. This allows to rephrase (iv) as: The global success probability of the route plan is higher than the reliability threshold.

The VRPHTWSST belongs to the broad family of stochastic Vehicle Routing Problems. Several sources of uncertainty and different solution approaches have been investigated in literature. Stochastic service and/or travel times have been considered for example in Kenyon and Morton (2003); Laporte et al. (1992); Lei et al. (2012); Tas et al. (2012) where either customer time windows are absent, or soft, or a maximal route duration is considered. Hard customer deadlines are considered in Campbell and Thomas (2008) where however the source of uncertainty is the customer presence. The VRPHTWSST addresses uncertainty by means of a probabilistic constraint. With respect to previous works, it considers a combination of (service) time uncertainty and customers hard time windows. To the best of our knowledge, such a problem has never been studied before.

We solve the VRPHTWSST by Branch & Price (B&P). In particular we provide a new set-covering formulation which includes a probabilistic constraint. The subproblem (SP) is solved via Dynamic Programming (DP). In order to reduce the number of considered states, we deploy new heuristic and exact dominance rules taking into account both route reduced cost and success probability. This is done by developing a recursive method to exactly compute the arrival time probability distribution at customers. Furthermore, adapting the method in Desaulniers et al. (2008), we speed up the algorithm by alternating DP and Tabu Search in SP. Preliminary result show that on modified Solomon's R100 and RC100 benchmark instances, our algorithm optimally solves all but two of the 50-customer instances.

2 Set-covering formulation with a probabilistic constraint

Let us consider a route r defined as a sequence of nodes $r = (v_0, v_1, \ldots, v_q, v_{q+1})$ where $v_1, \ldots, v_q \in V_c$ and v_0 and v_{q+1} represent the depot 0 and let $\mathcal{R}$ be the set of all possible routes. Let a_{ir} be a parameter with value 1 if route r visits customer i and 0 otherwise. The cost associated with a route r is $c_r = \sum_{i=0}^{q} c_{v_i, v_{i+1}}$. Considering binary variables x_r with value 1 if route $r \in \mathcal{R}$ is chosen and 0 otherwise, the VRPHTWSST can be formulated as follows:

$$\min \sum_{r \in \mathcal{R}} c_r x_r \tag{1}$$

$$s.t. \sum_{r \in \mathcal{R}} a_{ir} x_r \geq 1 \qquad \forall i \in V_c \tag{2}$$

$$\Pr\{ \text{ All chosen routes are successful} \} \geq \alpha \tag{3}$$

$$x_r \in \{0, 1\} \qquad\qquad \forall r \in \mathcal{R}, \qquad (4)$$

where the objective function (1) minimizes the total travel costs. Inequalities (2) assure that all customers are visited at least once. Observe that the assumption of non-negative costs implies that customers are visited exactly once. Inequality (3) represents the probabilistic constraint and assure that the success probability of the overall plan is higher than the reliability threshold. Relations (4) constrain the solution vector to be binary. The assumption that service times are mutually independent implies the property stated in the following proposition.

Proposition 2.1 *Let $\mathcal{R}'$ denote a set of routes inducing a proper partition of the customers set V_c. Given any two routes $r_1, r_2 \in \mathcal{R}'$, the success probability of r_1 is independent from the success probability of r_2.*

Setting $\beta_r := -\ln(\Pr\{ \text{ Route } r \text{ is successful } \})$ and $\beta := -ln(\alpha)$, Proposition 2.1 allows to rewrite constraint (3) as:

$$\sum_{r \in \mathcal{R}} \beta_r x_r \leq \beta. \qquad (5)$$

3 Methodology

We solve the VRPHTWSST by B&P, i.e., by embedding Column Generation (CG) in a Branch & Bound scheme. In CG a Restricted Master Problem (RMP) considers only a subset of the problem columns and is iteratively solved. At each iteration a subproblem (SP) searches for improving columns, i.e., columns with negative reduced costs. When such columns are found, they are added to RMP. Otherwise, the optimal solution of the continuous relaxation of the RMP of the current node has been found and branching can be performed.

Given a current RMP solution and dual multipliers $\gamma_i \geq 0$ and $\pi \geq 0$ associated with constraints (2) and (5) respectively, SP has to find a route minimizing the reduced cost $\bar{c}_r = c_r - \sum_{i \in V_c} a_{ir} \gamma_i + \beta_r \pi$. This is equivalent to finding a resource constrained shortest path where limited resources include the success probability. We solve such a problem by DP. In order to reduce the number of considered states, dominance rules allowing for implicit pruning of partial routes are a key element.

To briefly illustrate such dominance rules, consider two partial routes r_1 and r_2 visiting the same customer v_i and let us call $M_l(z)$ the probability that route l arrives at customer v_i at time $t \leq z$ provided that no failure occurred at previous customers, for $l = 1, 2$. The following proposition, whose proof is omitted due to space limitations, holds.

Proposition 3.1 *If $\bar{c}_{r_1} \leq \bar{c}_{r_2}$ and $M_1(z) \geq M_2(z)$ for all $a_{v_i} \leq z \leq b_{v_i}$, route r_1 dominates r_2 in the sense that any given extension of route r_1 will always imply lower cost and higher success probability than if the same extension was applied to r_2.*

A central issue for the VRPHTWSST is to efficiently compute the arrival time probability distributions at customers $M_l(z)$. In our algorithm, assuming discrete and independent service time distributions, we compute exactly the distributions $M_l(z)$ using a recursive method based on convolution of truncated probability distributions at previous customers, where truncations are induced by customer time windows. Hence no loss of information is incurred.

4   Results and Conclusions

In our preliminary experiments we considered suitably modified instances of the well known Solomon's data sets. In particular our instances preserve customers' location and time windows, while demand and vehicle capacity are ignored. We considered discrete service time distribution having symmetric support w.r.t the original value. In our preliminary experimentation we considered several discrete triangular distributions differing in the amplitude of the support as well as in their skewness. Preliminary results on instances derived from Solomon's R100 and RC100 classes show that our method optimally solves all the 25-customer instances and 18 of the 20 50-customer instances within 5 hours of computing time. In the presentation we will give details of our method and discuss the results of an extensive computational campaign.

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