A 3-phase method for a vehicle routing problem with stochastic travel and service times

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1 Introduction

The stochastic vehicle routing problem (SVRP), we are dealing with, is defined on a set of customers ($V = U \cup NU$) and a set K of m vehicles. Each customer $i \in U$ is a so-called urgent customer and must be visited during a hard time window $[e_i, l_i]$. Each customer in NU is non-urgent and may be visited during the planning period. A profit p_i is associated with $i \in NU$. Each vehicle $k \in K$ is based at the driver's home and is available during a limited period of time corresponding to working hours for the driver. The SVRP consists of determining a set of m vehicle routes of maximum profit, such that : i) each urgent customer is visited within the associated time window; ii) Each route starts from and ends at the driver's home within the planning period. Moreover, we assume that the service times and the travel times between customers are stochastic. A generic application of the SVRP is the design of routes for technicians. Urgent customers are requiring repair operations and are known a priori. Non-urgent customers are requiring a service (control, meter reading, maintenance, ...). At any time, a non-urgent customer can be postponed.

To the best of our knowledge, no previous work has been reported on the problem we just defined. Therefore, we focus here on variants of the vehicle routing problem with time windows (VRPTW) with common characteristics. Even if the VRPTW has been adressed in many articles, few are those taking simultaneously into account stochastic travel and service times. Wang et Regan (2001) formulate this problem as an assignment problem whereas Li et al. (2010) present a tabu search algorithm. In 2007, Zeimpekis et al. (2007) associate priorities with customers (depending on profit, on time window and on travel cost to access a customer) but consider the single vehicle case. In this framework, they propose a variant of the S-algorithm (Tsiligirides, 1984).

Regardless of the stochasticity, (Tricoire, 2006; Bostel et al., 2008, 2011) have been interested in the technician routing problem with time window, multiple depots and priority within customers : distinguishing urgent and non-urgent customers. They propose a column generation based method as well as a memetic algorithm. In 2006, Dugardin (2006) deals with the same problem but considers stochastic travel times. However, he does not take into account the stochasticity when building the route plan. Given a route plan, he defines simple rules to react to the different events which may occur. Recently, Borenstein et al. (2009) and Delage (2010) consider stochastic service times. Borenstein et al. (2009) proceed in many steps : they first partition customers into different clusters, then assign technicians to them. Finally, they make the borders of the technician zones fuzzy and they improve the current solution by allocating tasks of these fuzzy areas to adjacent areas. Delage proposes a two-step method : he first establishes a workload planning and then he deals with stochastic service times thanks to a dynamic programming approach. Last, Cortés et al. (2010) and Souyris et al. (2012) address the VRPTW with stochastic service times and priority within customers (corresponding to a target response time).

2 A 3-phase method

In the variant of the SVRP considered, all customers are known a priori. Travel and service times are stochastic but we assume that lower and upper bounds are known. As Delage (2010), we propose a three phase method : i) determine a skeleton defined on urgent customers ; ii) insert non-urgent customers in this skeleton ; iii) use dynamic programming to deal with the stochasticity on travel and service times.

2.1 Phase I : Skeleton design

As we do not allow any delay for serving urgent customers, we consider that travel and service times are maximal in this phase. Let $\overline{\sigma}_i$ be the maximal service time for customer i and $\overline{\tau}_{ij}$ be the maximal travel time between i and j. We define the following variables. A binary variable x_i^k indicates if urgent customer i is served by vehicle k. A binary variable y_{ij}^k indicates if customer i is served just before j by vehicle k, and last t_i corresponds to the time at which service starts at customer i. Then the skeleton design problem is as follows :

$$\min_{x,y,t} \sum_{i\in U}\sum_{j\in U} \overline{\tau}_{ij} \sum_{k\in K} y_{ij}^k \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in K} x_i^k = 1 \qquad \forall i \in U \tag{2}$$

$$\sum_{j \in U} y_{ij}^k = x_i^k \qquad \forall i \in U, k \in K \tag{3}$$

$$\sum_{i \in U} y_{ij}^k = x_j^k \qquad \forall j \in U, k \in K \tag{4}$$

$$e_i \leqslant t_i \leqslant l_i \qquad \forall i \in U \tag{5}$$

$$t_j \geqslant t_i + \overline{\sigma}_i + \overline{\tau}_{ij} + M \sum_{k \in K} (y_{ij}^k - 1) \qquad \forall i \in U, j \in U \tag{6}$$

$$x_i^k, y_{ij}^k \in \{0; 1\} \qquad \forall i \in U, j \in U, k \in K$$

Constraints (2) state that every urgent customer must be served exactly once. Constraints (3) and (4) are in-degree and out-degree constraints. Constraints (5) ensure the respect of time windows. Finally, constraints (6) are precedence constraints.

2.2 Phase II : Insertion of non-urgent customers

Given the skeleton, we tighten time window $[e'_i, l'_i]$ associated with urgent customer i. Then, we define the concept of *phalanx* : a phalanx is a route segment between two successive urgent customers. The origin depot and destination depot of each vehicle are considered as urgent customers. A phalanx has three main characteristics : an origin o^p, a destination d^p and a length $\Delta^p = l'_{d^p} - e'_{o^p} - \underline{\sigma}_{o^p}$ where $\underline{\sigma}_{o^p}$ is the minimum service time for customer o^p. Since we aim to insert as many non-urgent customers as possible in this second phase, we consider that travel and service times are minimal. Under this assumption, inserting non-urgent customers in the skeleton consists in building a path associated with each phalanx of the skeleton while ensuring for each phalanx p that the length of the path does not exceed Δ^p. The objective function in this step is double : first, maximize the profit associated with the visits of non-urgent customers, and then minimize the total traveled time. We decided to aggregate both objectives into a single one corresponding to a weighted sum of these two criteria. Then, we define α as the weight for the travel time and P as the set of phalanxes. Using the same notations as in phase I, and denoting the minimal travel times as $\underline{\tau}_{ij}$ and service times as $\underline{\sigma}_i$, we have the following model where variables x_i^p and y_{ij}^p are related to phalanxes and not to vehicles:

$$\max_{x,y} \sum_{p \in P} \sum_{i \in NU} p_i x_i^p - \alpha \sum_{p \in P} \sum_{i \in NU} \sum_{j \in NU} \underline{\tau}_{ij} y_{ij}^p \tag{7}$$

$$\text{s.t.} \quad \sum_{p \in P} x_i^p \leqslant 1 \qquad \forall i \in NU \tag{8}$$

$$\sum_{j \in NU \cup \{d^p\}} y_{ij}^p = x_i^p \qquad \forall i \in NU, p \in P \tag{9}$$

$$\sum_{i \in NU \cup \{o^p\}} y_{ij}^p = x_j^p \qquad \forall j \in NU, p \in P \tag{10}$$

$$\sum_{i \in NU \cup \{o^p\}} \sum_{j \in NU \cup \{d^p\}} \underline{\tau}_{ij} y_{ij}^p + \sum_{i \in NU} \underline{\sigma}_i x_i^p \leqslant \Delta^p \qquad \forall p \in P \tag{11}$$

$$\sum_{i\in S}\sum_{j\in S} y_{ij}^p \leqslant \sum_{i\in S\setminus\{l\}} x_i^p \qquad \forall S \subset NU, |S| \geqslant 2, \forall l \in S \tag{12}$$

$$x_i^p, y_{ij}^p \in \{0;1\} \qquad \forall i \in NU, j \in NU, p \in P$$

Constraints (8) state that each non-urgent customer is served at most once. Constraints (9) and (10) are in-degree and out-degree constraints. Constraints (11) enforce the length of the route on a phalanx to be smaller than the length of this phalanx. Constraints (12) are subtour elimination constraints.

2.3 Phase III : Dynamic programming

In this last step, we take into account the stochasticity on travel and service times and we modify the route planning in real-time. We consider the vehicles one by one and we assume that phalanxes are sorted the following way : phalanx $p+1$ has for origin urgent customer $o^{p+1} = d^p$. Each dynamic programming step corresponds to the end of service at a customer's location. At each step, we have a list of non-urgent customers who may be served before the next urgent customer and two options have to be considred : either the driver goes directly to the next urgent customer or he visits the non-urgent customer in this list who maximizes the profit.

At step k, let v_k be the customer served, t_k the end of service time for this customer, V^p be the list of non-urgent customers associated with the phalanx and $\bar{V}_k^p$ customers among V^p that have not been served at step k.

We propose two different dynamic programming types. In the first one, we consider only one phalanx and the revenue function can be stated as follow :

$f(v_k, t_k, \bar{V}_k^p) = \max(E[f(d^p, t_k + \tau_{v_k d^p}, \emptyset)], 1 + \max_{\bar{v}\in\bar{V}_k^p} E[f(\bar{v}, t_k + \tau_{v_k\bar{v}} + \sigma_{\bar{v}}, \bar{V}_k^p\setminus\{\bar{v}\})])$

with $f(d^p, t, \emptyset) = -\alpha_{d^p}\max(t - l_{d^p}, 0)$

In the second one, we consider the remaining route. The revenue function is slightly different, namely :

$f(v_k, t_k, \bar{V}_k^p) = \max(E[\hat{f}(d^p, t_k + \tau_{v_k d^p})], 1 + \max_{\bar{v}\in\bar{V}_k^p} E[f(\bar{v}, t_k + \tau_{v_k\bar{v}}, \bar{V}_k^p\setminus\{\bar{v}\})])$

with $\hat{f}(d^p, t) = -\alpha_{d^p}\max(t - l_{d^p}, 0) + f(d^p, t + \sigma_{d^p}, V^{p+1})$

2.4 Improvements

When few urgent customers have to be visited, this method may lead to obtain solutions where vehicles are unused while many non-urgent customers remain unserved. To avoid this pitfall, we balance workload over vehicles during the insertion of non-urgent customers. We decompose phase II into two stages : first, insert non-urgent customers with pessimistic estimations of travel and service times and then insert non-urgent customers with optimistic estimations of travel and service times while keeping the assignment of non-urgent customers to vehicles obtained in the first stage.

For the insertion of non-urgent customers, we develop a branch-and-cut algorithm which includes four main features to speed up the solution process, namely : a heuristic to build an initial solution, some preprocessing on variables, replacement of subtour elimination constraints by "Reachability Cuts" (Lysgaard, 2006) and addition of valid subset elimination inequalities : let S be a subset of customers and $L^p(S)$ be the length of the shortest path from o^p to d^p associated with phalanx p and serving all customers in S. If $L^p(S) > \Delta^p$, then the subset elimination inequality $x^p(S) \leqslant |S| - 1$ is a valid constraint.

3 Computational results

To evaluate our method, we extracted daily instances from real-life instances proposed by Tricoire (2006). The two first phases are solved exactly using Cplex 12.4 in a branch-and-cut framework. The last phase is combined to a simulator to generate random travel and service times. The machine, we used for our experiments, has four 3.2Ghz CPUs and 1Go of RAM. In the following table, the computation times in seconds are reported for the 3-phase method where the insertion of non-urgent customers consists in a single step (the one with pessimistic estimations). Different variants of the branch-and-cut algorithm are considered. The columns headings are the followings: B&C : Branch and Cut (with branching priorities on x variables); P : Preprocessing on variables; H : Heuristic to generate an intial solution; RC : Reachability Cuts; SEI : Valid subset elimnation inequalities.

4 Conclusions

We proposed a 3-phase method for solving a SVRP with stochastic travel and service times. It consists in designing a skeleton on urgent customers, inserting non-urgent customers in this skeleton and last using a dynamic programming approach to face stochasticity on travel and service times. Since the simplest algorithm requires large CPU times for solving small instances, we proposed four major improvements (preprocessing on variables, a heuristic to generate an initial solution, "Reachability Cuts" and valid subset elimination inequalities). Thanks to these improvements, all instances with up to 36 customers are solved within less than 7 minutes. When larger instances are considered, most of them remain unsolved within 2 hours. Thus, some improvements are still required. However, in our method, we assume that there is a sufficient number of urgent customers (otherwise, the skeleton design would be useless). Therefore, a more promising research avenue should be to merge the first two phases of our method into a single one.

Instances	Nb of cust.	B&C	B&C, P	B&C, P, H	B&C, P, H, RC	B&C, P, H, RC, SEI
C4_10	4U, 14NU	1	1	2	1	1
C4_11	6U, 19NU	3	3	2	1	2
C4_12	6U, 20NU	3	3	3	3	2
C4_13	7U, 21NU	1746	53	61	36	22
C4_14	6U, 19NU	3	2	2	2	2
C4_20	7U, 21NU	233	28	26	22	16
C4_21	7U, 21NU	336	29	26	22	16
C4_22	8U, 24NU	> 7200	378	379	265	131
C4_23	8U, 24NU	> 7200	378	379	265	131
C4_24	7U, 21NU	56	12	9	17	11
C4_30	6U, 20NU	7522	22	22	15	11
C4_31	7U, 23NU	4129	543	148	52	27
C4_32	8U, 26NU	>7200	>7200	>7200	1067	421
C4_33	9U, 27NU	>7200	1750	2275	337	161
C4_34	7U, 22NU	>7200	2399	752	118	32
C4_40	6U, 18NU	89	10	10	10	8
C4_41	7U, 23NU	>7200	193	279	57	46
C4_42	9U, 27NU	>7200	656	653	272	92
C4_43	8U, 25NU	>7200	600	675	205	86
C4_44	7U, 21NU	1441	38	32	17	14
C4_50	6U, 20NU	87	7	7	8	6
C4_51	8U, 24NU	>7200	143	178	92	47
C4_52	7U, 23NU	173	14	13	31	22
C4_53	8U, 24NU	>7200	672	649	296	94
C4_54	6U, 20NU	36	13	7	10	10

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