PATH-DIFFERENTIATED PRICING FOR CONGESTION MITIGATION

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1 INTRODUCTION

Price differentiation or discrimination is a situation in which identical products are sold at different prices (see Dupuit, 1894) and there are three degrees of price differentiation (Pigou, 1932). In first-degree differentiation, the price of a product equals to the buyer's maximum willingness-to-pay. For the second-degree, the price depends on the number of units to be purchased. Finally, the price of a product in third-degree differentiation varies with the types or characteristics of the buyers.

This talk views different paths connecting the same origin-destination (OD) pair as being the same product because they serve the same OD pair. Price differentiation occurs when different paths serving the same OD pair are charged or tolled at different prices. Specifically, path tolls in this talk are used to encourage motorists to use routes that lead to a target link flow distribution, e.g., one that minimizes total delay. A set of path tolls is valid if they induce or, to use an economic vernacular, decentralize a target distribution. Additionally, possible sets of valid path tolls are infinite and a secondary objective is often used to select one for implementation. In this talk, the secondary objective is to minimize toll revenue in order to lessen the financial burden on motorists.

When compared to link tolls (i.e., tolls charged for usage of individual roads or links) path tolls are more flexible because paths are more numerous than individual links. Adding together link tolls on a given path also yields a toll for the path. So, a set of valid link tolls always induces a set of valid path tolls. The converse is generally false.

Being more flexible, path tolls typically impose less financial burden on motorists in that they permit less revenue to be collected than link tolls. This talk views the reduction in toll revenue that path tolls offer as a way of appraising the monetary value of the information concerning travel routes. Performing this appraisal requires finding link and path tolls that generate the least revenue. When every link is tollable and all tolls are nonnegative, Hearn and Ramana (1998) show that finding valid link tolls with the minimum revenue can be formulated as a linear program, a problem relatively easy to solve. Finding path tolls with minimum revenue, on the other hand, is more difficult and can be formulated as mathematical program with complementarity or equilibrium constraints. However, alternative formulations exist and some are more advantageous computationally.

2 TOLL PRICING AND BENEFITS FOR PATH INFORMATION

To illustrate the benefits of using path tolls, consider the four-node network in Figure 2.1. Nodes are labeled as o, a, b, and d. Links are numbered from 1 to 7 as indicated by the subscript of the link performance function adjacent to each link. There is only one OD pair, (o,d), and its demand is five. Table 2.1 provides the target link distribution $\bar{x}$ and the minimum-revenue (MINREV) link tolls, β^{mr}, that generate a revenue of 10 units (see the last row).

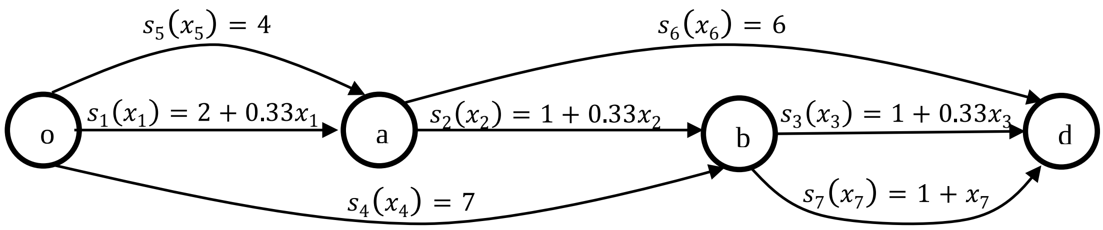

Figure 2.1: Four-node network.

For the above network, there are only eight possible paths for the OD pair and they are listed in the second column of Table 2.2 below. When aggregated, the path distribution in the third column of Table 2.2 supports or yields the target link distribution $\bar{x}$ in Table 2.1. Observe that only the first five paths are utilized. The MINREV path tolls are in the column labeled π^{mr}. These path tolls are valid because the generalized cost of every utilized path is the same (9 units) and no larger than those not utilized. Thus, users have no incentive to switch to paths 6, 7 and 8. As shown the last row in Table 2.2, π^{mr} generates five units of revenue. So, path tolls in this example reduce the revenue by 50% (from 10 units to 5) when compared to link tolls.

Table 2.1: MINREV link tolls for the four-node network

Link	$\bar{x}_a$	$s_a(\bar{x}_a)$	β_a^{mr}
1	3	3	1
2	3	2	1
3	3	2	1
4	1	7	0
5	1	4	0
6	1	6	0
7	1	2	1
Revenue			**10**

Table 2.2: MINREV path tolls for the four-node network

Path	Links in path	f_p	$\bar{c}_p$	π_p^{mr}	$\bar{c}_p + \tilde{\pi}_p$
1	1 → 2 → 3	1	7	2	9
2	5 → 2 → 3	1	8	1	9
3	4 → 3	1	9	0	9
4	1 → 2 → 7	1	7	2	9
5	1 → 6	1	9	0	9
6	5 → 2 → 7	0	8	2	10
7	4 → 7	0	9	2	11
8	5 → 6	0	10	2	12
Revenue				**5**	

3 MINIMUM REVEUNE TOLL PRICING PROBLEM

As mentioned previously, Hearn and Ramana (1998) show that finding link tolls with the minimum revenue (or the link-toll pricing problem) can be formulated as a linear program. However, the corresponding problem with path tolls has not been explored. Our literature survey discovered only one journal article (Dafermos, 1973) with a substantive treatment of path tolls. In her paper, Dafermos offers a set of feasible link tolls involving the familiar marginal cost (see, e.g., Arnott and Small, 1994, Lindsay and Verhoef, 2001, and Yin and Lawphongpanich, 2009) and adds up the tolls on links along each path to obtain feasible path tolls in a setting with multiple classes of users.

We initially formulate the path-toll pricing problem as a mathematical program with equilibrium constraints. Because the latter is a difficult optimization problem to solve, the pricing problem is also reformulated as mixed integer programs (MIP-1 and MIP-2), concave and bilinear minimization problems. Both of the latter are beyond the scope of this talk. However, the two integer programs can be solved by commercial software. MIP-2 offers promising computation results on road networks from the literature. When compared to

MIP-1 (a more natural formulation of the pricing problem), we suspect that MIP-2 is stronger in that its relaxation offers a tighter approximation of the original problem.

During the talk, we plan to illustrate and discuss the benefits associated with path tolls other than lowering the financial burden. In addition, we will explore alternative formulations for the path-toll pricing problem mentioned above, investigate their properties, and provide results from our experiments with MIP-2 and data from the road networks in Anaheim and Sioux Falls.

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