On a Theory for Potential Capacity Gains due to Connected and Autonomous Trains

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1 INTRODUCTION

This paper continues the work carried out in "Fundamental diagrams and emergent dynamics of mainline rail operations" (Morey *et al.*, 2021) and explores modelling improvements in capacity that could be available from the digitalisation of rail operations by increasing levels of the European Train Control System (ETCS) (Stanley *et al.*, 2011). Technological advances in communications and receiving data (e.g., via 5G radio-based systems) may enable real-time monitoring and analysis (Theeg & Vlasenko, 2020), which could help increase line capacity, via use of moving-block signalling. In particular, trains might share position and speed directly with their immediate neighbours, which might achieve better line capacity by implementing what we call train-following models (TFMs) —– analogous to the car-following models (CFMs) that are used to describe highway traffic.

2 SUMMARY OF LEGACY FIXED-BLOCK THEORY

In Morey *et al.* (2021), we investigated the line capacity of mainline rail operations that use legacy fixed-blocked signalling rules, summarised by Pachl (2020) and the multi-author volume Theeg & Vlasenko (2020). In summary, the safety-critical principles are that (i) track is partitioned into blocks; (ii) each block should only be occupied by at most one train at any time; (iii) each train must be able to come to a stop safely within the blocks ahead (known as the 'aspects') that its driver knows to be clear, see Figure 1(a); and (iv) each train must be able to come to a stop safely ahead of any block ahead that its driver knows to be occupied by another train, see Figure 1(b).

Constant acceleration rules were then applied to Figure 1(a) to derive a maximum safe speed

$$v_{\max}^2 := 2b(\alpha L_{\mathrm{B}} - L_{\mathrm{M}}), \tag{1}$$

where b is the braking rate, α is an integer-valued aspect parameter, L_{B} is block length (which for simplicity is assumed the same for all blocks), and $L_{\mathrm{M}} \ll L_{\mathrm{B}}$ is a safety margin.

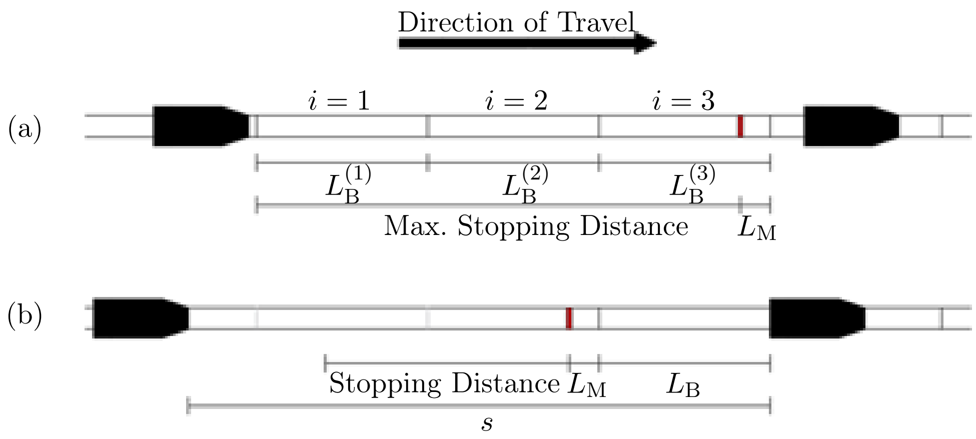

Figure 1 – *Safety limit cases: represents $\alpha = 3$ (four aspect) fixed block signalling, where (a) the follower train must be able to stop safely within the blocks it knows are free, also (b) net spacing, s, must be sufficient for the follower to come to a stop before the leader's block.*

In contrast, constant acceleration formulae applied to Figure 1(b) yield a bound for speed

$$v^2 \leq 2b(s - L_B - L_M), \tag{2}$$

which involves the net-spacing s. Relation (2) is analogous to the equilibrium (uniform-flow) speed-spacing relationship in a CFM (Wilson, 2001). It thus forms the basis for deriving a set of fundamental diagrams (FDs) that describe smooth running scenarios via relations between speed, density, and flow (corresponding to timetabled frequency). Trade-offs between the various parameters were examined in some detail by Morey *et al.* (2021). The starting point for this analysis is to write $\rho := 1/(s + L_T)$ (where L_T is the train length) and to employ a jam density $\rho_J := 1/(L_T + L_B + L_M)$ to find the speed-density relationship

$$v = V(\rho) := \min\left[v_\dagger \left(\frac{1-\tilde{\rho}}{\tilde{\rho}}\right)^{1/2}, v_{\max}\right], \tag{3}$$

where $v_\dagger := (2b/\rho_J)^{1/2}$ and $\tilde{\rho} := \rho/\rho_J$, with $0 \leq \tilde{\rho} \leq 1$, defines a re-scaled density.

The resulting FDs are summarised in Figure 2 (black lines), with parameter choices $L_T = 400\text{m} < L_B = 1600\text{m}$, $L_M = 100\text{m}$, $\alpha = 2$, $b = 0.65\,\text{ms}^{-2}$ and $a = 0.4\,\text{ms}^{-2}$, justified in Morey *et al.* (2021).

3 CONNECTED AND/OR AUTONOMOUS TRAINS

Our new work is to consider potential capacity improvements achievable by the introduction of connected and/or autonomous trains (CATs). We assume that these CATs share each others' positions and speeds directly and do not have to obey fixed-block rules. Moreover, they are considered to be omniscient, rendering the limit described in Figure 1(a) and (1) redundant. In contrast, the effect of continuous position knowledge is to set the block length L_B to zero. Thus (3) is modified by setting $v_{\max} = \infty$ (although other practical physical limits might apply instead) and by using an increased jam density $\rho_J := 1/(L_T + L_M)$. This gives rise to a new set of FDs indicated by the blue lines in Figure 2, quantifying the capacity gains. In summary, at any given spacing (alternatively, density), the CATs will be faster than legacy trains. In consequence, flow is increased — the parameters we have chosen suggest that a doubling of capacity is possible.

4 CONNECTION TO GIPPS'S CAR-FOLLOWING MODEL

The modelling so far has focused on safe operations under constant braking assumptions. This is also the premise of the classical Gipps CFM (Gipps, 1981), which (translating into the notation

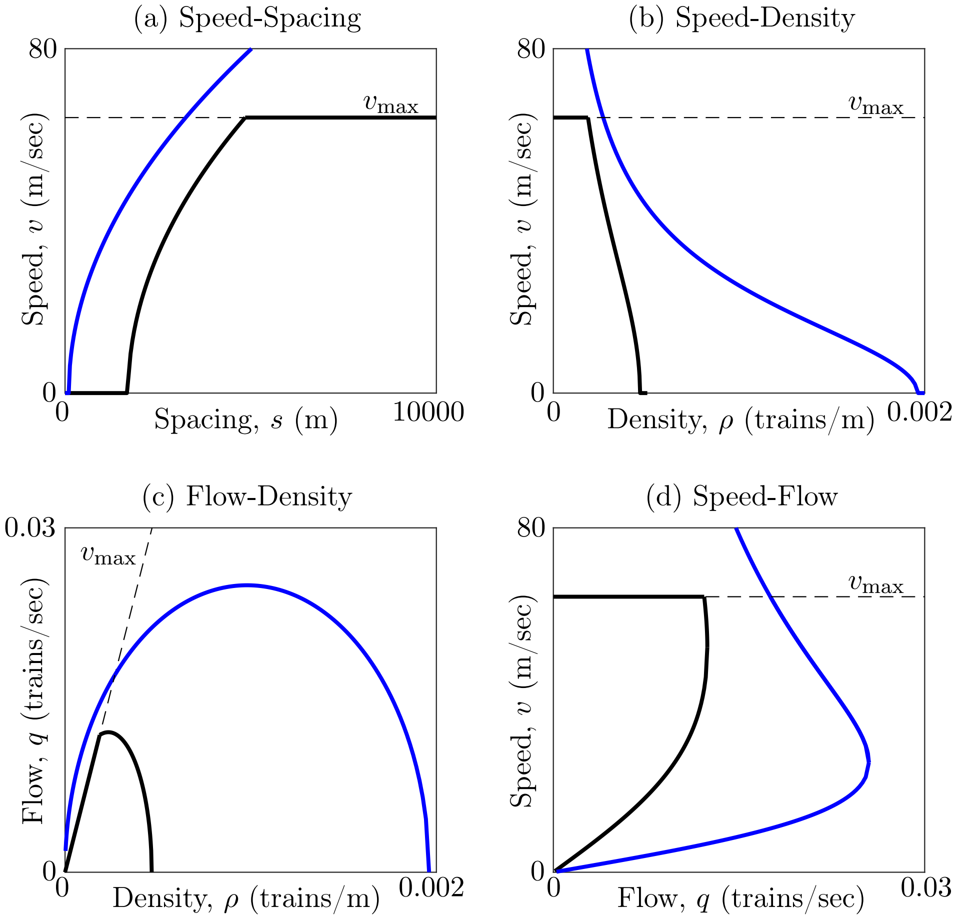

Figure 2 – *Fundamental diagrams (FDs) for mainline rail traffic. Black lines show legacy operations and blue lines show the improved performance of connected and/or autonomous trains. (a) Speed v versus net spacing s. (b) Speed v versus density ρ. (c) Flow q versus density ρ. (d) Speed v versus flow q.*

of this paper) has an equilibrium speed-spacing function (Wilson, 2001) that takes the form

$$v = \left(\frac{\tau+\theta}{\frac{1}{b}-\frac{1}{\hat{b}}}\right)\left[-1+\sqrt{\left\{1+\frac{2s\left(\frac{1}{b}-\frac{1}{\hat{b}}\right)}{(\tau+\theta)^2}\right\}}\,\right]. \tag{4}$$

Here $\hat{b}$ is the maximum assumed braking rate of the lead vehicle, and τ and θ are parameters that model a safety margin in terms of reaction-time delays. For trains, we effectively assume the worst case scenario where the leader may undergo a 'brick-wall' stop. Hence we set $\hat{b} = \infty$. Exploiting definitions of $v_\dagger$ and ρ_J, we may thus obtain

$$v = -v_\ddagger + \left[v_\ddagger^2 + v_\dagger^2\left(\frac{1-\tilde{\rho}}{\tilde{\rho}}\right)\right]^{\frac{1}{2}}, \tag{5}$$

where $v_\ddagger := b(\tau+\theta)$ describes the additional velocity reduction that might have been achieved if there were no reaction-time delay. By setting $v_\ddagger = 0$, we recover the earlier result (3).

We might use (5) a starting point to model the reduction in capacity due to communication and actuation delays. However, supposing that $\tau+\theta$ is of the order of several seconds, then $v_\ddagger$ is of the order of just 1 or 2 ms^{-1}, far less than typical mainline operating speeds $v_\dagger$. So the effect of these delays on capacity is likely to be rather small.

There are a couple of further avenues for investigation. Firstly, (i) Gipps's model (4) with $\hat{b}$ finite would allow us to relax the 'brick-wall' stop assumption of the earlier theory — although this seems at odds with current railway safety thinking. Secondly (ii) it is known (Morey *et al.*, 2021, Wilson, 2001) that these Gipps-based models can display dynamical instabilities and other undesirable behaviour. Finally (iii) one should consider mixtures of legacy trains — one might say Driver Operated and/or Guided trains (DOGs) — and CATs. However, the benefits of CATs are only realised if the CATs follow other CATs — which might be very difficult to organise given practical timetabling constraints. We have a proverb in English: it is very difficult to herd CATs!

Of course, line capacity is only one part of the broader question of operational rail capacity, which also involves node capacity, fleet utilisation, crew provision etc. These points will be expanded upon in our presentation at the TRISTAN meeting.

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