Polynomial approximate dynamic programming scheme for the weight constrained two-echelon routing problem with a drone

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1 Introduction

In this abstract, we investigate a generic variant of a 2-echelon routing problem, in which the travel range of the primal vehicle (2^{nd} echelon vehicle) is constrained by one or several capacitated resources. By meeting at the same place and the same time with the supporting vehicle (1^{st} echelon vehicle) for a *service* in one of possible *meeting points (MPs)*, those resources can be replenished. The arising problem is characterized by particularly strong synchronization constraints for the two vehicles compared to the well-studied 2-echelon problem formulations. The described problem setting emerges in many robot routing applications, notably, those including a drone. In the latter case, the supporting vehicle may be a truck (Poikonen & Golden, 2020) or an autonomous mobile replenishment station (Barrett *et al.*, 2018). In possible applications, the drone, constrained by its data storage capacity, collects data from scattered sensors (Jawhar *et al.*, 2014); or it delivers packages, the total weight of which can not exceed its payload (Poikonen & Golden, 2020); or it visits several locations for monitoring in a long mission that surpasses its flight range (Baik & Valenzuela, 2021). The current pandemics gave a further boost to the deployment of drones, which now spray disinfectants in public spaces, pick up lab samples, and deliver tests and medical supplies (UNICEF, 2021).

This paper reports on selected results of a larger research project on the described 2-echelon routing problem, which we dubbed as *the Drone Routing Problem with Energy replenishment (DRP-E)*. The project is conducted by the authors and Nicola Mimmo (University of Bologna).

DRP-E is a generalization of several routing problems, such as the *asymmetric traveling salesman problem with replenishment arcs* (Boland *et al.*, 2000), the *traveling salesperson problem with hotel selection* (Castro *et al.*, 2015), and *drone routing with stationary service stations* (Mathew *et al.*, 2015). Contrary to the available literature on drone routing applications, such as *traveling salesman problems with drone* (Agatz *et al.*, 2018), DRP-E allows the drone to visit multiple destinations between consecutive meetings for a service and permits both vehicles to revisit arcs and MP's. Furthermore, most studied drone routing scenarios consider simplified energy consumption which is linear in the flight time. To the best of our knowledge, only Poikonen

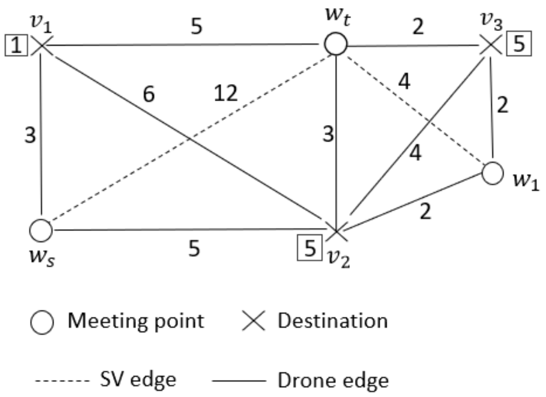

Figure 1 – *An instance of DRP-E with one resource (energy) and load-dependent energy consumption*
Instance with $|V^d| = 3$ and $|V^m| = 3$. The tour must start in w_s and end in w_t. The edge weights mark the distances, the (boxed) node weights denote the package weight. The drone's energy consumption depends on the flight time t and the current load p: $E(t,p) = t\cdot(p+1)$. The drone's energy capacity is $c_{max} = 55$.

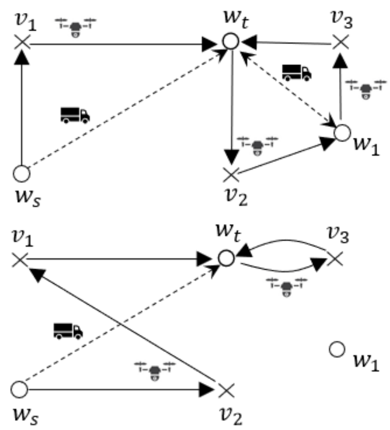

Figure 2 – *Solutions for the instance of Figure 1*
Top: Solution of the route-first-cluster-second approach RTS based on TSP path $(w_s, v_1, v_2, v_3, w_t)$. The tour consists of 3 operations (w_s, v_1, w_t), (w_t, v_2, w_1) and (w_1, v_3, w_t) of duration 12, 5 and 4 respectively. The total makespan is 21.
Bottom: Optimal solution with 2 operations (w_s, v_2, v_1, w_t), (w_t, v_3, w_t) of duration 16 and 4, respectively. The total makespan is 20, or 5% less than in the solution above.

& Golden (2020) consider the problem setting described by DRP-E; the authors propose a route-first-cluster-second construction heuristic, which they dub *RTS*. This state-of-the-art approach first fixes the visiting order of the required destinations by the primal vehicle as a TSP-path, then optimally selects and inserts MPs into this fixed sequence of destinations.

Similarly to RTS, many state-of-the art approaches for 2-echelon routing problems exploit the separability of the routing decisions for both echelons. Those provide good starting solutions, but are hard to improve by standard local search operators such as k-exchanges, swaps or randomized neighbourhood searches, and usually show a large gap to optimality. This gap generally grows with the addition of realistic problem features that strengthen the interdependence of both vehicles. In our project, we propose a *(polynomial approximate) dynamic programming scheme (PADP)* that considers routing on both echelons simultaneously and is able to significantly improve the initial solution, such as that of RTS, in reasonable running time. By dropping the aggregation of the states in the state graph, PADP can be straightforwardly transformed into a powerful *exact* dynamic programming approach.

2 Problem description

In DRP-E, the primary vehicle, which we call *drone*, has to visit a given set of *destinations* V^d. Because of the limited capacity of one or several resources, the drone has to meet regularly with the *supporting vehicle (SV)* in a set of potential *meeting points (MP)*, denoted as V^m, which are safe spots on the SV's road network for servicing the drone. A service usually implies a battery swap, but may also refer to data transmissions to the data storage, package pick-ups, or package deliveries. Both vehicles depart from the source $w_s \in V^m$ and terminate their trip in the target $w_t \in V^m$. We call the walks of both vehicles in between two consecutive meetings for a service at MP's w and w' as *operation (OP)* $o = wsw'$, where s is the corresponding sequence of destinations visited by the drone. The duration of an OP wsw' is determined by the latest arrival of the two vehicles at w' (synchronization) and possibly involves waiting times. An OP is feasible if the consumption of each resource $r \in R$ in the corresponding drone walk does not exceed its respective capacity $c^r_{max} \in \mathbb{R}$. The resource consumption functions are generally complex and do not solely depend on the flight time. Each mission can be considered as a sequence of feasible operations.

The objective of DRP-E is to minimize the makespan of the drone's mission, by:

- sequencing the destination visits of the drone (2^{nd} echelon)
- scheduling the services, which requires routing the SV through selected MP's (1^{st} echelon)

Figure 1 illustrates an instance of DRP-E with one resource (energy), in which the energy drain depends on the load carried by the drone.

3 Contributions and computational results

The developed solution scheme PADP stands out from conventional heuristics in a number of aspects which will be elaborated in detail during the presentation:

- The proposed PADP explores an exponential number of promising solutions in polynomial time. This is due to a novel state-space aggregation procedure and an efficient encoding of the states. The path exploration in the aggregated state graphs optimizes *simultaneously* two polynomial subproblems of DRP-E. Given an input sequence of destinations, the first subproblem compares similar missions, in which the visiting order of destinations verifies the precedence constraints defined by Balas & Simonetti (2001). The second polynomial subproblem takes the visiting order of destinations as given and optimizes, when and where the services take place. The simultaneous solution of the mentioned subproblems by PADP exploits synergies and significantly boosters the efficiency of the solution procedure.
- PADP can be straightforwardly adapted to many 2-echelon routing problem variants. This flexibility arises from the design of PADP as a two-phase approach: The first phase constitutes of $|V^m|$ weight-constrained one-to-many shortest path problems, which identify feasible operations and eliminate dominated ones. The associated aggregated state-graph, dubbed *operations graph*, accommodates all drone-related features and is searched by a label setting algorithm. In the second phase, these pre-selected feasible operations are sequenced to a best possible complete mission. The second phase accommodates the specificity of the drone service and the characteristics of the SV. Note that the two-phase approach does not separate the routing decisions for different echelons.
- The intensity of the aggregation in the state graphs of PADP is flexibly scaled by only one parameter p, which controls the trade-off between the computational complexity and accuracy. By dropping the aggregation, PADP – which is exponential in p, but polynomial in the parameters of DRP-E – can be straightforwardly transformed to an *exact* dynamic programming approach, which is the first exact solution approach for DRP-E so far.

We *analytically* estimate the size of the explored solution space and the time complexity of PADP. Then, we evaluate the performance of PADP in extensive computational tests on various structured artificial benchmark data sets. On the basis of different application scenarios, we identify which factors reinforce or deteriorate the benefits of PADP compared to the state-of-the-art heuristics. Table 1 shows an extract from the conducted experiments. Benchmark instances are generated as described in Poikonen & Golden (2020) for a quadcopter operating in a square area of *length* l (km) with zero package weights (NoPack) and package weights between 0 and 2.3 kg (Pack). PADP outperformed the state-of-the-art procedure RTS in almost all the instances. In several cases the observed improvement exceeded 10%. Observe that RTS-solutions cannot be easily improved even after hundreds of iterations with standard local search operators neither applied to the route-first part of the solution, nor to the resulting mission, as we show in our extensive experiments. Each instance with $|V^d| = 16$ destinations and $|V^m| = 16$ MPs was solved by the *exact* algorithm based on PADP on a standard laptop with Intel i7-8565U, 1.80 GHz, 32GB RAM within a 10-minute time limit with an average runtime of 193 seconds.

We also examine real-life case studies, such as a search-and-rescue operation by a drone and an autonomous robotic charging platform in the mountainous areas around lake Occhito, in Southern Italy. In the mentioned case study, PADP outperformed the state-of-the-art solution in disaster relief by 20%.

Table 1 – *Performance of PADP*

Setting ($\|V^d\|$–$\|V^m\|$–l–P)	Avg. (worst) gap to optimality (%)	Avg. (best) improve- - ment over RTS (%)	# improvements over RTS
16-16-20-noPacks	2.1 (6.7)	1.4 (5.4)	5 (out of 10)
16-16-20-Packs	3.6 (9.7)	3.9 (7.5)	10 (out of 10)
25-25-25-noPacks	– (–)	2.7 (7.6)	10 (out of 10)
25-25-25-Packs	– (–)	5.5 (13.8)	10 (out of 10)
50-25-25-noPacks	– (–)	3.7 (8.6)	10 (out of 10)
50-25-25-Packs	– (–)	5.8 (9.7)	10 (out of 10)
25-50-25-noPacks	– (–)	2.1 (7.6)	10 (out of 10)
25-50-25-Packs	– (–)	4.5 (8.4)	10 (out of 10)
Total	– (–)	3.7 (13.8)	75 (out of 80)

4 Conclusion

We currently examine PADP in a number of further real-world case studies. As next steps, the initial problem formulation as well as our analysis and the developed solution scheme PADP should be extended to further cases relevant for practice. These include problem formulations with uncertain flight times and multiple drones.

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