# Risk evaluation of a real-time Railway Traffic Management solution under uncertain dwell times 

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## 1 INTRODUCTION

The real-time Railway Traffic Management Problem (rtRTMP) is the problem of detecting and solving time overlapping conflicting requests done by multiple trains on the same track resources in order to compute a new plan of operations (Cacchiani et al., 2014). Typically, the rtRTMP is modelled as a deterministic problem and requires to be solved in a short computation time. As such, both the structural and numerical model elements are often assumed to be fixed and known in advance. However, uncertainty may still affect the computed plans and their expected quality, since operation times may vary due to a change in a dynamic environment. This work proposes a new approach to evaluate the risk of delays associated to an rtRTMP solution when operation times are uncertain and only an interval representation of them is known to the scheduler.

Among the operating times we focus on the so called dwell times, which represent the amount of time spent by trains on a platform to allow passengers boarding/alighting. They have been shown in the literature to have a significant effect on network capacity and are recognized as one of the main source of uncertainty (Larsen et al., 2014), over which the operators may have little or no control and influence. The design, test, and implementation of advanced mathematical models is a prerequisite to the development of innovative decision support systems for solving the rtRTMP (Borndörfer et al., 2017, Pellegrini et al., 2019). Following this path, this work considers the possible uncertainties that may affect a solution for rtRTMP, and proposes a new method to compute a measure of the risk of delays.

In fact, the reliability of a scheduling solution is important for dispatchers to make informed decisions, especially when operating in real-time. A tool able to quantify the risk associated to a schedule should offer a practical support to decision-makers, especially a tool that evaluates the delay risks of a schedule taking advantage of a real-time forecasting of dwell times, assuming that they are represented by prediction intervals (Kecman \& Goverde, 2015). The decision makers may be risk-averse and may prefer solutions that do not just perform well "on average", but that also perform satisfactorily "in most cases".

Adopting a coherent risk measure is a way of modeling risk in scheduling problems. To this aim, a popular coherent risk measure is the Expected Shortfall (ES) or Conditional-Value-at-Risk (CVaR) (e.g., see (Bertsimas et al., 2004, Meloni \& Pranzo, 2020, Sarin et al., 2014)) which measures the average loss in the worst $(1-\alpha) 100 \%$ of outcomes, with $\alpha \in[0,1]$. This loss can represent a delay in railway transport services. This type of risk measure emphasizes the contribution of the worst-case outcomes under the index performance of interest. $\mathrm{CVaR}_{\alpha}$ is recommended when a decision maker is not only concerned with the frequency of undesirable outcomes, but also with their severity (Bertsimas et al., 2004, Sarin et al., 2014).

In this work we propose an approach that deals with the evaluation of the CVaR for the delay associated to rtRTMP solutions when the practical realization of the integer valued duration of operations are uncertain at the scheduling stage and may take any value within a given interval (i.e., between a lower and an upper bound obtained from an available prediction system) (Meloni \& Pranzo, 2020). More precisely, we consider real-time uncertainty limited to dwell times and address the evaluation of the CVaR associated to a feasible schedule exploiting a graph model representation of the problem to take advantage of a real-time information collection system. To this aim, extending the approach recently proposed by (Meloni et al., 2021) for more general activity networks, we develop and test a suitable computational method to evaluate the CVaR of the delay of a given rtRTMP solution. The proposed method enables the use of this risk index as evaluation criterion for different railway scheduling approaches.

## 2 METHODOLOGY

We propose a risk aware rtRTMP solution framework as a computational tool for solving the rtRTMP and assessing the risk in terms of possible delays associated with the obtained solutions on the basis of the information received from a dwell time prediction system. The rationale is to provide, through the proposed framework, the dispatcher with a restricted pool of rtRTMP solutions for their final choice and approval in view of the actual implementation.

Firstly, the framework receives as input an instance of rtRTMP which is solved through a deterministic scheduler on the basis of available nominal data, corresponding to the minimum duration values. In our computational analysis, we model the rtRTMP as Job Shop Scheduling Problem using the Alternative Graph model and considering as objective function the minimization of the maximum delay $M_{d}$ introduced in the plan by scheduling decisions. We then solve the rtRTMP using AGLibrary, a state-of-the-art optimization solver that deals with complex routing and scheduling problems (D'Ariano et al., 2007, Samà et al., 2017). The solver continues providing further rtRTMP solutions until a stopping criteria is reached. In our implementation, the stopping criteria is either the rtRTMP solution optimality, or the reaching of the maximum allowed computation time.

Each time a new rtRTMP solution is found, it is translated into a corresponding temporal activity network with integer interval-valued durations (IIN) (Meloni \& Pranzo, 2020) which includes the independent dwell times prediction intervals. Specifically, an IIN can be described by the pair $\left(G^{\prime}, \mathbf{D}\right)$, where $G^{\prime}$ is a directed network of $r$ activities, and $\mathbf{D}=\left(\mathbf{D}_{1}, \ldots, \mathbf{D}_{r}\right)$ is a vector of independent integer interval durations associated with the $r$ activities. The considered network is directed, connected, and acyclic with single source and sink nodes. Interval activity integers times $T_{p}=\left[a_{p}, \overline{a_{p}}\right]$ for $p=1, \ldots, r$ are assigned to the activity durations $\mathbf{D}=\left(\mathbf{D}_{1}, \ldots, \mathbf{D}_{r}\right)$ representing their specific number of time units, with $a_{p}$ and $\overline{a_{p}}$ integers, and $a_{p} \leq \overline{a_{p}}$ for all $p$. When a duration value of an activity $p$ is known with certainty, we have the deterministic case represented by an interval $T_{p}$ where $a_{p}=\overline{a_{p}}$. We observe that, even if this study focuses on the dwell time uncertainties, this modeling solution is quite general and can be used to represent interval-valued uncertainties associated with all the arcs in $G^{\prime}$.

The IINs have an intrinsically discrete nature, and for a given IIN network $Q$ the uncertain performance measure $\mathbf{M}_{d}$ is finite and bounded keeping values in $\left[\underline{M_{d}}, \overline{M_{d}}\right]$, where the optimistic
value $\underline{M_{d}}$ (the pessimistic value $\overline{M_{d}}$ ) refers to the configurations of $Q$ in which all durations are at their minimum (maximum) value. The finite length $\Gamma$ (in terms of number of integers) of the time interval $\left[\underline{M_{d}}, \overline{M_{d}}\right]$ represents a measure of the amount of uncertainty of the IIN. The CVaR at probability level $\alpha$ of $\mathbf{M}_{d}$ in the IIN is adopted as risk measure, and it is finite and bounded holding the following: $\underline{M_{d}} \leq \mathrm{CVaR}_{\alpha}\left(\mathbf{M}_{\mathbf{d}}\right) \leq \overline{M_{d}}, \forall \alpha \in[0,1]$.

The IIN forms the input of the framework module devoted to the risk delay evaluation which is based on the $\operatorname{CVaR}_{\alpha}\left(\mathbf{M}_{\mathbf{d}}\right)$ index. As for this risk measure, one or more probability levels $\alpha$ may be chosen by the dispatchers to represent different distributions of the effects of (possible) extreme realizations. Higher values of $\alpha$ are of interest for risk-averse decision makers, while $\alpha=0$ can be associated with the risk-neutral behavior. In fact, when selecting values of $\alpha$ tending to 1 the $\operatorname{CVaR}_{\alpha}\left(\mathbf{M}_{\mathbf{d}}\right)$ tends to the pessimistic case $\overline{M_{d}}$; whereas for values of $\alpha$ tending to $0, \mathrm{CVaR}_{\alpha}\left(\mathbf{M}_{\mathbf{d}}\right)$ tends to the expected value $E\left(M_{d}\right)$ of the adopted delay index.

All rtRTMP solutions found by the solver are processed for risk delay assessment then are collected and analyzed in a solution archive that can be further exploited in order to identify a restricted pool of preferable solutions, e.g., adopting some additional criteria. This allows the dispatchers to be aware of quality and risk of a feasible solution and thus to choose the one that solves the trade-off between performance level and the risk measures they are most comfortable with, possibly also in consideration of company protocols or risk regulations.

## 3 RESULTS

To show how the risk aware rtRTMP solution works, we consider an example instance based on the railway network around the central station of Utrecht (NL), the busiest station in the Netherlands. Of the 140 existing dwell time constraints, $50 \%$ randomly chosen are affected by uncertainty. Specifically, we assume as deterministic duration for each dwell time its minimum value. For each dwell time arc $p$ affected by uncertainty, its interval duration is $T_{p}=\left[\underline{a_{p}}, \overline{a_{p}}\right]$, where $\overline{a_{p}}=\delta a_{p}$, and with $\delta \in\{1.25,1.50,1.75,2.00\}$ to gradually represent the severity of the uncertainty. The time unit adopted is 1 second. Regarding the probability levels for the CVaR assessment, three different values are used, i.e., $\alpha \in\{0.99,0.95,0.90\}$, meaning the assessment of the average of $M_{d}$ for the worst $1 \%, 5 \%$, and $10 \%$ cases. The solver found 6 feasible solutions for the example instance, taking 0.2 seconds, with an optimal solution value of $M_{d}^{*}=105 \mathrm{~s}$. A total of 72 risk evaluations was carried out for all solutions found. The risk assessment takes 0.24 s on average.

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\alpha \times 0.99 \times 0.95 \times 0.90
$$



Figure 1 - Example instance with $50 \%$ of uncertain dwell times and $\delta \in\{1.25,1.50,1.75,2.00\}$.
Figure 1 shows the risk analysis for all feasible solutions found on the 4 uncertain scenarios. Each scenario has a dedicated plot, labeled with the related $\delta$ value. The x-axis represents
the maximum delay value $M_{d}$ associated with the deterministic solution found. The y-axis represents the risk evaluation of a feasible solution with respect to the uncertainty affecting it, i.e. the $\operatorname{CVaR}_{\alpha}\left(\mathbf{M}_{\mathbf{d}}\right)$ index, in terms of how much the solution may worsen. We represent in the same plot the evaluations made with different $\alpha$ values using a colored cross symbol as reported in the legend on top. In a given scenario, considering probability levels $\alpha_{1} \leq \alpha_{2}$, for a feasible solution $\operatorname{CVaR}_{\alpha_{1}}\left(\mathbf{M}_{\mathbf{d}}\right) \leq \operatorname{CVaR}_{\alpha_{\mathbf{2}}}\left(\mathbf{M}_{\mathbf{d}}\right)$ holds. The bold vertical axis at $x=105$ indicates the value of the deterministic optimal solution.

This analysis allows the decision maker to easily identify (if any) the presence of feasible solutions with better risk profiles than the optimal deterministic solution. For example, in the scenario with the lowest level of uncertainty $(\delta=1.25)$ there are no such cases. While, as the level of uncertainty increases, various alternative solutions are revealed. Here, two types of cases are visible. The first one represent solutions that offer a better risk profile, yet having a worse deterministic value. In the figure this is shown by the crosses that are not on the bold vertical line, i.e., the $M_{d}$ deterministic optimal value, but have lower values of CVaR. These cases call the decision maker to resolve a trade-off. The second case, on the other hand, happens when more than one solution exists with optimal deterministic value, though they differ for their risk profiles, as clearly visible for $\delta=1.75$ and $\delta=2.00$, where on the bold vertical line two solutions with the same optimal value but different CVaR are visible. The dominant one is preferable.

As the degree of uncertainty increases, solutions show a different risk profile which also tends to vary more significantly on the basis of the $\alpha$ levels. In these cases, the decision maker's risk attitude becomes relevant.

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