A Multi-directional Aggregated Model for Large Urban Networks

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1 Introduction

The development of traffic models has been in its majority influenced by the model structure proposed in the the Lighthill-Whitham-Richards model (LWR) Lighthill & Whitham (1955) and Richards (1956). This model is the most simple and therefore the most popular macroscopic model for traffic. Many variants of this model consider traffic flow that has only one direction of motion. The first attempt to include multiple directions in 2D continuum models was made only several years ago, when Lin *et al.* (2017) took inspiration from pedestrian modeling and deployed dynamic user-optimal principle for the path choice. The drawback of this model is that the traffic density may become unbounded. There exist also other works Mollier *et al.* (2019), Mollier *et al.* (Jan. 2019), Aghamohammadi & Laval (2019) proposing 2D multi-layer models with bounded densities. However, they do not include traffic mixing between different direction layers. These models are also not necessarily hyperbolic, i.e., their PDE type varies with parameters, which exaggerates its analysis and numerical simulation. Hyperbolicity for all parameters implies that it can be analysed like many other conservation law based models for traffic.

In this work we propose a new multi-direction traffic flow model called the NEWS model. This macroscopic model is composed by a set of four partial differential equations (PDEs), each modelling the density propagation in one of the four cardinal directions: North, East, West and South. The model is formally derived from the classical cell transmission model at intersections. Knowledge about the network topology (location of the roads) and network infrastructure parameters (roads maximal speeds, number of lanes and capacities) are used to tune the model. The information about the flow direction is retrieved from the turning ratios at the intersections, which is then aggregated in four directions using projection matrices. The paper also discusses some of the mathematical properties of the model, namely, hyperbolicity and mass conservation law. We will show some trials for the model validation, using synthetic data from a microsimulator mimicking the Grenoble downtown network. In addition, the model is also validated using real data collected from real sensors installed in Grenoble.

2 The NEWS model

The NEWS model describes the multi-directional evolution of vehicle density on a bounded 2D continuum plane $\Omega \subset R^2$ that is a rectangular domain bounded by x_{min}, x_{max}, y_{min} and y_{max}.

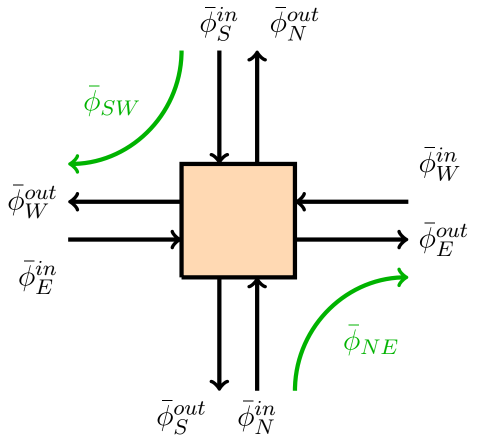

Figure 1 – *Schematic explanation of flow directions in NEWS formulation.*

It is given by the following set of partial differential equations $\forall (x, y, t) \in \Omega \times R^2$:

$$\begin{cases} \dfrac{\partial \bar{\rho}_N}{\partial t} = \dfrac{1}{L}\left(\bar{\phi}_N^{in} - \bar{\phi}_N^{out}\right) - \dfrac{\partial(\overline{\cos\theta_N}\bar{\phi}_N)}{\partial x} - \dfrac{\partial(\overline{\sin\theta_N}\bar{\phi}_N)}{\partial y}, \\ \dfrac{\partial \bar{\rho}_E}{\partial t} = \dfrac{1}{L}\left(\bar{\phi}_E^{in} - \bar{\phi}_E^{out}\right) - \dfrac{\partial(\overline{\cos\theta_E}\bar{\phi}_E)}{\partial x} - \dfrac{\partial(\overline{\sin\theta_E}\bar{\phi}_E)}{\partial y}, \\ \dfrac{\partial \bar{\rho}_W}{\partial t} = \dfrac{1}{L}\left(\bar{\phi}_W^{in} - \bar{\phi}_W^{out}\right) - \dfrac{\partial(\overline{\cos\theta_W}\bar{\phi}_W)}{\partial x} - \dfrac{\partial(\overline{\sin\theta_W}\bar{\phi}_W)}{\partial y}, \\ \dfrac{\partial \bar{\rho}_S}{\partial t} = \underbrace{\dfrac{1}{L}\left(\bar{\phi}_S^{in} - \bar{\phi}_S^{out}\right)}_{\text{mixing term}} - \underbrace{\dfrac{\partial(\overline{\cos\theta_S}\bar{\phi}_S)}{\partial x} - \dfrac{\partial(\overline{\sin\theta_S}\bar{\phi}_S)}{\partial y}}_{\text{transportation term}}. \end{cases}$$

The state of this model is given by the four-dimensional vector $\bar{\rho} = \bar{\rho}(x, y, t) = (\bar{\rho}_N, \bar{\rho}_E, \bar{\rho}_W, \bar{\rho}_S)$, representing densities of the traffic flows in four direction layers. Functions $\bar{\phi}(x, y, \bar{\rho})$, $\bar{\phi}^{in}(x, y, \bar{\rho})$ and $\bar{\phi}^{out}(x, y, \bar{\rho})$ are defined via fundamental diagrams and are space- and state-dependent. Finally, $L(x, y)$, $\overline{\cos\theta(x, y)}$ and $\overline{\sin\theta(x, y)}$ are *space-dependent* parameters which aggregate the topology of a particular traffic network.

The right-hand side of the model consists of two parts: transportation term and mixing term. The transportation term consists of spatial derivatives and thus describes the transportation of traffic density in the same way as it is done in the original LWR model. Therefore, the NEWS model can be seen as a **continuous generalisation of LWR to a 2-dimensional multi-directional urban traffic networks**. The mixing term accounts for coupling between traffic flows in different direction layers. In particular, $\bar{\phi}^{in}(x, y, \bar{\rho}) - \bar{\phi}^{out}(x, y, \bar{\rho})$ is the net flow at roads:

$$\begin{pmatrix} \bar{\phi}_N^{in} - \bar{\phi}_N^{out} \\ \bar{\phi}_E^{in} - \bar{\phi}_E^{out} \\ \bar{\phi}_W^{in} - \bar{\phi}_W^{out} \\ \bar{\phi}_S^{in} - \bar{\phi}_S^{out} \end{pmatrix} = \begin{pmatrix} \bar{\phi}_{EN} + \bar{\phi}_{WN} + \bar{\phi}_{SN} - \bar{\phi}_{NE} - \bar{\phi}_{NW} - \bar{\phi}_{NS} \\ \bar{\phi}_{NE} + \bar{\phi}_{WE} + \bar{\phi}_{SE} - \bar{\phi}_{EN} - \bar{\phi}_{EW} - \bar{\phi}_{ES} \\ \bar{\phi}_{NW} + \bar{\phi}_{EW} + \bar{\phi}_{SW} - \bar{\phi}_{WN} - \bar{\phi}_{WE} - \bar{\phi}_{WS} \\ \bar{\phi}_{NS} + \bar{\phi}_{ES} + \bar{\phi}_{WS} - \bar{\phi}_{SN} - \bar{\phi}_{SE} - \bar{\phi}_{SW} \end{pmatrix}.$$

Thereby $\bar{\phi}_{EN}$, $\bar{\phi}_{WN}$, $\bar{\phi}_{SN}$ and so on are *partial flows*, e.g., $\bar{\phi}_{SN}(x, y, \bar{\rho}_S, \bar{\rho}_N)$ is a flow of cars that were going to the South and then turned to the North. This information should be retrieved from turning ratio measurements. Figure below illustrates the concept of partial flows on an example of one intersection.

This simple analytic representation of urban traffic can facilitate the analysis for explicit control design, as it was shown in Tumash *et al.* (2021). Details on the model derivation and the particular flow functions can be found in Tumash *et al.* (2021).

(a) Fixed sensors

(b) Bluetooth devices

Figure 2 – *Sensor location in Grenoble downtown: (a) fixed flow sensors: R denote radars and L denote induction loops; (b) automatic vehicle identifiers using Bluetooth installed at* 12 *intersections of Grenoble during a measurement campaign lasting for one week. These figures are taken from Rodriguez-Vega (2021).*

3 Model validation with real data

The model was validated with real data, using the Grenoble Traffic Lab experimental platform, see `http://gtlville.inrialpes.fr/`. The platform collects real-time traffic data from Grenoble downtown as shown Fig. 2. The sensor measure boundary flows (from radars and loops) and turning ratios at some intersections. Boundary flows are used as a flow boundary condition in the model. Turning ratios are used in the model as a parameters. The boundary flows are also used separately to estimated the internal density of the network, to be compared with the one predicted by the model.

The maximal densities $\rho_{max,j}$, capacities $\phi_{max,j}$, road lengths l_j and orientations θ_j are computed from the network topology. In addition, and to complete the model we also use average velocities obtained from a data provider. Finally, the method for estimating: the free-flow speed, the full set of turning ratios, the vehicle density and the boundary flows are described in more details in Rodriguez-Vega *et al.* (June 2021).

In Fig. 2a) the sensors marked in blue are those giving boundary inflows and red sensors give boundary outflows. Sensors marked in green were used for the validation of state estimation procedure.

The results are depicted in Fig. 3, where the comparison of two density distributions is shown. We see that in both cases the distributions look quite similar. Part of the differences between the model predicted density evolution, may come from: the NEWS model does not include traffic lights, and does not take into account parking lots. Another source of mismatch could be induced by data on inflows and outflows, namely the data represents measured flows in the city that we can not enforce in our model, since there is always a demand-supply problem that not always match with the actual measure. The source code used for model validation is an *open source* project that can be found here: `https://github.com/Lyurlik/multidirectional-traffic-model`.

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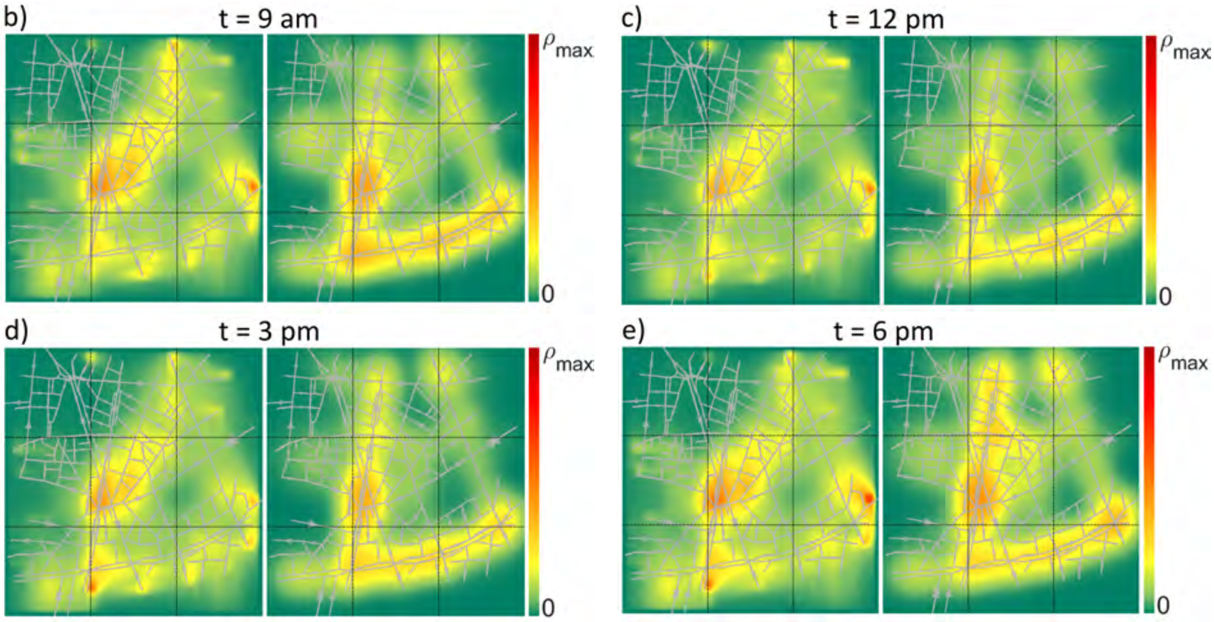

Figure 3 – *Evolution of traffic density in Grenoble downtown on* 8*th of January* 2021 *from* $t = 6$ *am to* $t = 9$ *pm: numerical simulation of NEWS model (left plots) and density estimated from sensor data (right plots). Weighting parameter* $\eta = 20$. *These figures are taken from* Tumash *(2021)*

References

Aghamohammadi, R., & Laval, J. A. 2019. A Continuum Model for Cities Based on the Macroscopic Fundamental Diagram: a Semi-Lagrangian Solution Method. *Transportation Research Procedia*, **38**, 380–400.

Lighthill, J. M., & Whitham, G. 1955. On kinematic waves, II: A theory of traffic flow on long crowded roads. *Proc. R. Soc. Lond. A*, **229**, 317–345.

Lin, Z. Y., Wong, S. C., Zhang, P., Jiang, Y. Q., Choi, K., & Du, Y. C. 2017. A predictive continuum dynamic user-optimal model for a polycentric urban city. *Transportmetrica B: Transport Dynamics*, **5**(3), 228–247.

Mollier, S., Delle Monache, M.L., Canudas-de-Wit, C., & Seibold, B. 2019. Two-dimensional macroscopic model for large scale traffic networks. *Transportation Research Part B: Methodological*, **122**, 309 – 326.

Mollier, S., Delle Monache, M. L., & Canudas-de-Wit, C. Jan. 2019. A step towards a multidirectional 2D model for large scale traffic networks. *Pages 1–7 of: TRB 2019 - 98th Annual Meeting Transportation Research Board, Washington D.C., USA.*

Richards, P. 1956. Shock waves on the highway. *Operations Research*, **47**(1), 42–51.

Rodriguez-Vega, M. 2021. *Optimal sensor placement and density estimation in large-scale traffic networks.* University Grenoble Alpes: PhD Thesis.

Rodriguez-Vega, M., Canudas-de-Wit, C., & Fourati, H. June 2021. Urban network traffic state estimation using a data-based approach. *In: CTS 2021 - 16th IFAC Symposium on Control in Transportation Systems, Lille, France.*

Tumash, L. 2021. Traffic Control in Large-Scale Urban Networks. *PhD from the Grenoble University Alpes, France.*

Tumash, L., Canudas-de-Wit, C., & Delle Monache, M.-L. 2021. Boundary Control for Multi-Directional Traffic on Urban Networks. *IEEE 60th Conference on Decision and Control (CDC), December 2021, Austin, Texas, USA.*

Tumash, L., Canudas-de-Wit, C., & Delle Monache, M.L. 2021. Multi-Directional Continuous Traffic Model For Large-Scale Urban Networks. *to appear in Transportation Research Part B: Methodological.*