An Improvement MIP model for Routing Problems

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1 Introduction

Vehicle routing problems are well-studied within the field of transportation science, and they exist in a variety of versions with numerous solution methods. Three relevant versions are the capacitated vehicle routing problem (CVRP), the inventory routing problem (IRP) and the production routing problem (PRP). In the CVRP, a set of customers with a known demand for a product needs to be served by a fleet of vehicles with inventory capacities. The vehicles start at a depot which they return to at the end of the day. All customers must be visited and all demands must be fulfilled. In the IRP, there is a time horizon consisting of several days. Each customer has a storage of the product which they consume over time. The customers' storage is refilled by the producer and transported by a set of vehicles and customers are not allowed to experience stock-outs. The aim is to minimize both inventory costs and routing costs. The PRP is similar to the IRP, but here we control the production at the depot where the vehicles pick up the product as well. We refer to the surveys of Laporte (2009) and Adulyasak *et al.* (2015) for more details on the CVRP, IRP and PRP.

Most vehicle routing problems have in common that they are very hard to solve. One way to model vehicle routing problems is to introduce a route set $\mathcal{R}$ consisting of every possible route. Here, a route is just a string of customer nodes. We then choose which routes in $\mathcal{R}$ we should use in order to solve the problem. We call this a path-flow formulation. However, we can replace $\mathcal{R}$ with a much smaller route set $\mathcal{R}'$ and still obtain the optimal solution as long as $\mathcal{R}'$ includes the optimal routes. Vadseth *et al.* (2021) and others have used this successfully in matheuristics. However, finding the correct routes is not trivial and small alterations from the optimal routes can lead to low-quality solutions.

Hence, to overcome the aforementioned challenges we introduce a mixed integer programming (MIP) model to help us solve routing problems with a route set $\mathcal{R}'$, but where we can make smaller changes to the routes in addition. We call this an improvement MIP model. The route set $\mathcal{R}'$ can either consist of the routes from a previously found solution or multiple solutions. We have used this improvement MIP model as part of a matheuristic for both the IRP and the PRP (Vadseth *et al.*, 2022) and to improve the best-known solution for several known benchmark instances for the CVRP. The improvement MIP model can also easily be adapted to other vehicle routing problems.

2 An Improvement MIP Model

The improvement MIP model presented here is a modified version of a path-flow model for the CVRP with a set of routes $\hat{\mathcal{R}}$. Here, the model has the possibility of modifying the chosen routes by inserting or removing customers. The improvement MIP model calculates the exact costs of altering the original routes and does so even in cases where multiple changes are made simultaneously. The presented improvement MIP model for the CVRP is similar to the model for the PRP presented in Vadseth *et al.* (2022). Please note that Improvement MIP is reduced to an IP for the CVRP.

We start by assuming that no customers are visited by a route and then insert the customer that should be visited. The variable y_r is 1 if route r is used, and 0 otherwise. In addition, the variable x_{ir} is 1 if node i is inserted into route r, and 0 otherwise. The cost of using route r is denoted C_r^R, and the cost of inserting node i into route r is denoted C_{ir}^N. The parameter Q is the vehicle capacity and D_i is customer i's demand. In addition, $\mathcal{N}'$ is the set of all customer nodes.

$$\min \sum_{r \in \hat{\mathcal{R}}} C_r^R y_r + \sum_{i \in \mathcal{N}'} \sum_{r \in \hat{\mathcal{R}}} C_{ir}^N x_{ir}, \tag{1}$$

$$\sum_{i \in \mathcal{N}'} D_i x_{ir} \leq Q y_r, \qquad r \in \hat{\mathcal{R}}, \tag{2}$$

$$\sum_{r \in \hat{\mathcal{R}}} x_{ir} = 1, \qquad i \in \mathcal{N}', \tag{3}$$

$$x_{ir} \leq y_r, \qquad i \in \mathcal{N}', r \in \hat{\mathcal{R}}, \tag{4}$$

$$y_r \in \{0, 1\}, \qquad r \in \hat{\mathcal{R}}, \tag{5}$$

$$x_{ir} \in \{0, 1\}, \qquad i \in \mathcal{N}', r \in \hat{\mathcal{R}}. \tag{6}$$

The objective function (1) minimizes the transportation costs, while constraints (2) ensure that a vehicle cannot deliver more than its maximum capacity. In addition, constraints (3) make sure that a customer can only be visited by at most one vehicle. The fact that a customer cannot be added to an unused route is taken care of by constraints (4). Constraints (5)-(6) define the variable domains. In addition, several instances have requirements regarding the minimum or maximum number of vehicles used.

To keep the transportation cost in the improvement MIP model correct with respect to the cost in the original problem, we introduce additional constraints to limit the number of changes that can be made to a route, and also on the type of changes. Without these limitations, some changes might look beneficial in the improvement MIP model but are actually disadvantageous to the solution of the original CVRP. To present these constraints some additional notation is introduced. The cost of traveling between nodes i and j is denoted C_{ij}.The original route r includes P^r customer visits, and we use $p = 0$ and $p = P^r + 1$ to denote the depot at the start and end of r. The set of customers visited by the original route r is denoted $\mathcal{N}^r$. The function $i_r(p)$ returns the node placed in position p in route r. Moreover, the function $p_r(i)$ returns the position of node i in route r if i was originally in r. If i was not originally in r however, $p_r(i) = argmin(C_{i_r(p-1),i} + C_{i,i_r(p)} - C_{i_r(p-1),i_r(p)} | p = 1, \ldots, P^r + 1)$ returns the position of a greedy insertion of i into r. Lastly, we introduce the parameter A_{ipr} which is 1 if node i is placed or would be greedily placed in position p in r, and 0 otherwise. Figure 1 illustrates the notation. The variable x_{0r} is defined to be equal to 1.

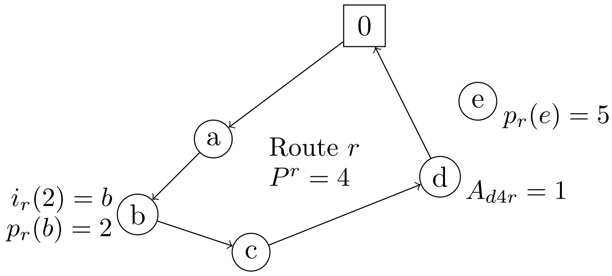


Figure 1 – *Illustration of a route r, the functions $i_r(p)$ and $p_r(i)$ and the parameter A_{ipr}.*

The following constraints are added to the formulation to ensure that the transportation cost is correctly calculated.

$$x_{i_r(p_r(i)-1),r} + x_{i_r(p_r(i)),r} \geq 2x_{ir} \qquad r \in \hat{\mathcal{R}}, i \in \mathcal{N}' \setminus \mathcal{N}^r \tag{7}$$

$$\sum_{i \in \mathcal{N}' \setminus \mathcal{N}^r} A_{ipr} x_{ir} \leq 1 \qquad r \in \hat{\mathcal{R}}, p \in 1, \dots, P^r + 1 \tag{8}$$

$$x_{ir} + x_{i_r(p_r(i)+1),r} \geq y_r \qquad r \in \hat{\mathcal{R}}, i \in \mathcal{N}^r \tag{9}$$

Constraints (7) state that if a node i that was originally not in route r is inserted, then we must include the nodes that are located before and after the position where node i is inserted into r. Constraints (8) ensure that at most one node, not originally in route r, can be inserted into the same position in r. Constraints (9) make sure that if route r is used, then either node i or its successor, which are both in $\mathcal{N}^r$, must be included in the route.

To calculate the insertion cost C^N_{ir} for node i in route r the following formulas are used:

$$C^N_{ir} = C_{i_r(p_r(i)-1),i} + C_{i,i_r(p_r(i))} - C_{i_r(p_r(i)-1),i_r(p_r(i))} \qquad r \in \hat{\mathcal{R}}, i \in \mathcal{N}' \setminus \mathcal{N}^r \tag{10}$$

$$C^N_{ir} = C_{i_r(p_r(i)-1),i} + C_{i,i_r(p_r(i)+1)} - C_{i_r(p_r(i)-1),i_r(p_r(i)+1)} \qquad r \in \hat{\mathcal{R}}, i \in \mathcal{N}^r \tag{11}$$

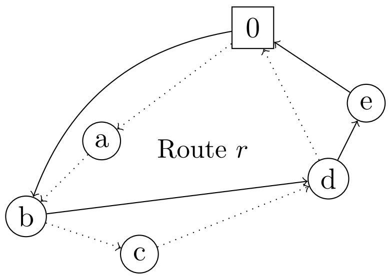
The route cost, C^R_r, for route r is given by the formula:

$$C^R_r = C^T_r - \sum_{i \in \mathcal{N}^r} C^N_{ir} \tag{12}$$

Here, C^T_r is defined as the cost of the original route without any changes. Hence, both the insertion cost and the route cost depends on the nodes included in the original route. Figure 2 shows the cost calculations of a route in the improvement model. The original route is marked with dotted arrows. After solving the improvement model, route r is included in the solution, but changes have been made to it. The new route is marked with solid arrows.

3 Results

The improvement MIP model has been tested as an essential part of a matheuristic for the PRP and IRP. The matheuristic was tested on two sets of benchmark instances for the PRP from the literature. The first set was introduced by Archetti *et al.* (2011) and consists of smaller instances, while the larger set was introduced by Boudia *et al.* (2007). For the Archetti instances, the matheuristic found the best-known solution for 516 out of 960 instances. Further, for the Boudia instances, the matheuristic found the best-known solution for 75 out of 90 instances. On the large multi-vehicle benchmark instances for the IRP released by Archetti *et al.* (2012) the method found the best-known solution for 93 out of 240 instances. See Vadseth *et al.* (2022) for more details.



$$C_r^T = C_{0a} + C_{ab} + C_{bc} + C_{cd} + C_{d0}$$
$$C_{ar}^N = C_{0a} + C_{ab} - C_{0b} \qquad C_{br}^N = C_{ab} + C_{bc} - C_{ac}$$
$$C_{cr}^N = C_{bc} + C_{cd} - C_{bd} \qquad C_{dr}^N = C_{cd} + C_{d0} - C_{c0}$$
$$C_{er}^N = C_{de} + C_{e0} - C_{d0}$$
$$\begin{aligned} C_r^R &= C_r^T - C_{ar}^N - C_{br}^N - C_{cr}^N - C_{dr}^N \\ &= C_{0b} + C_{ac} + C_{bd} + C_{c0} - C_{ab} - C_{bc} - C_{cd} \end{aligned}$$
$$\begin{aligned} 0 \rightarrow b \rightarrow d \rightarrow e \rightarrow 0 : &\ C_r^R + C_{br}^N + C_{dr}^N + C_{er}^N \\ &= C_{0b} + C_{bd} + C_{de} + C_{e0} \end{aligned}$$

Figure 2 – *Example showing the cost calculations when* b*,* d *and* e *are included in route* r *and* a *and* c *are excluded. The route cost* C_r^R *is calculated based on the original route, the cost of choosing route* r *and include* b*,* d *and* e *is shown to be correct.*

For the CVRP, the improvement MIP model was tested on the large benchmark instances released by Arnold *et al.* (2019) and where the route set $\hat{\mathcal{R}}$ was made up by the routes from the best-known solution. The improvement MIP model improved the best-known solution of five out of ten instances, and at a later stage six out of ten instances. See `http://vrp.atd-lab.inf.puc-rio.br/index.php/en/` for more details.

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