Branch-Price-and-Cut for the Electric Vehicle Routing Problem with Nonlinear Charging Function

G. M. Nafstad[a,*], G. Desaulniers[b] and M. Stålhane[a]

[a] Norwegian University of Science and Technology, Trondheim, Norway
gaute.m.nafstad@ntnu.no, magnus.staalhane@ntnu.no
[b] Polytechnique Montreal and GERAD, Montreal, Canada
guy.desaulniers@gerad.ca
* Corresponding author

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1 Introduction

The world's fleet of vehicles powered by electricity is growing. Important actors, such as the European Union, have indicated that they will start to phase out petrol vehicles in order to contribute to less greenhouse gas emissions. Several countries signed an agreement to stop sales of new emitting cars by 2040 during the 2021 United Nations Climate Change Conference.

Most of the electric vehicles today store energy in batteries. Battery and battery recharging technology are in rapid development, but driving range and recharging times remain well known drawbacks of electric vehicles. Route planning for electric vehicles introduces the need to account for a limited driving range and the possibility of recharging during a route. Petrol vehicles also have limited driving range, however, as time needed to refuel a petrol vehicle is negligible and the availability of gas stations is usually very high, it can be excluded from the route planning. The classical problem of routing petrol vehicles falls in the category of vehicle routing problems (VRP), while route planning with electric vehicles are referred to as the electric vehicle routing problem (E-VRP). The VRP is the problem of routing an unlimited fleet of homogeneous vehicles to visit a set of customers. Each customer must be visited exactly once by a vehicle. The vehicles start and end their route at a central depot. The objective is to minimize the distance traveled by the vehicles. In E-VRP, the vehicles have a given battery capacity which gives a limited range before having to either recharge or end the route. The objective is to minimize the total time spent on the routes, including travel time and recharging time. In addition to the depot and the customer nodes, the E-VRP has dedicated recharging nodes. The recharging process can be modelled in various ways, which is the main difference in several versions of the E-VRP. The goal of this paper is to present a Branch-Price-and-Cut (BP&C) solution method for the E-VRP with nonlinear charging functions (E-VRP-NL). The novelty of the method lies in the way recharging is handled in the pricing problem. The method is tested on the benchmark instances from Montoya *et al.* (2017) and solves several previously unsolved instances to optimality.

The E-VRP has received increasing attention in the last decade. The green vehicle routing problem, introduced by Erdoğan & Miller-Hooks (2012), was among the first to consider dedicated

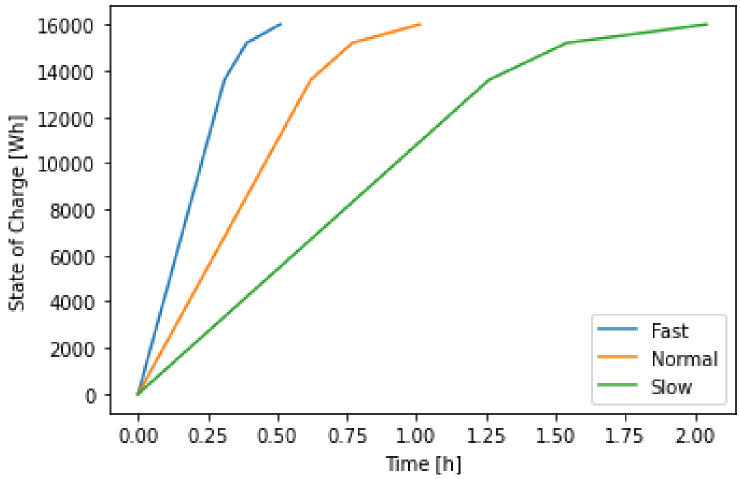

Figure 1 – *The recharging functions are the ones used in the Montoya benchmark instances (Montoya* et al.*, 2017).*

recharging locations into the VRP. In subsequent years, many extensions of the problem with different attributes have been contributed to the E-VRP literature. A survey by Erdelić & Carić (2019) presents many of these extensions and the tailored solution methods used, both exact methods and heuristics. Multiple of these variations relate to the modeling of the recharging procedure. Early works modelled the recharging process as a constant time procedure, similar to the process of battery swapping. In the works of Conrad & Figliozzi (2011), vehicles can charge a fixed percentage during a fixed recharging time at certain customer nodes. Later, other recharging modelling alternatives have been developed to better mimic the real recharging possibilities and the related recharging times.

One of these modelling possibilities is to allow for partial recharging of the vehicle, contrary to full recharging. This feature will increase the solution space, and hence might give improved solutions. Another feature that affects the size of the solution space is the limit on the number of visits to recharging nodes for a route. Some limit recharging to maximum once per route, while others have no upper limit. A discussion related to this potential limit, its importance when modelling, and its effect on the optimal solution is presented by Montoya *et al.* (2017).

Desaulniers *et al.* (2016) present a BC&P method for the E-VRP with time windows and linear charging, first introduced by Schneider *et al.* (2014). The method presented handles all four combinations that can be made by having full or partial recharging and one or multiple recharge operations for a route. The recharging function that describes the state of charge (SoC) as a function of time is linear in their case. In reality, the recharging function is a nonlinear concave function. The recharging function is known to be close to linear on the interval from 0 % SoC to about 80 %, and then the recharging rate decreases exponentially the remaining 20 % (Montoya *et al.*, 2017). Montoya *et al.* (2017) argue that a piecewise linear function with three segments is a sufficiently good approximation. They introduce the E-VRP with nonlinear charging function (E-VRP-NL). An additional feature in that problem is that the recharging nodes might have different recharging technologies, meaning that they do not necessarily have the same recharging function. The work also presents a set of benchmark instances for the problem. In these instances there are three types of recharging nodes; Fast, Normal and Slow. The piecewise linear approximations are shown in Figure 1 for a vehicle with a battery capacity of 16 kWh.

Montoya *et al.* (2017) solved the E-VRP-NL with a hybrid metaheuristic and presented a mixed-integer linear programming (MILP) formulation for the problem. Two tighter MILP formulations, one based on arc flow and another on path flow, are presented by Froger *et al.* (2019). That is, to the best of our knowledge, the only papers describing exact solution methods for the E-VRP-NL. Froger *et al.* (2021) presents a matheuristic for the E-VRP-NL with capacitated charging stations. They also test their algorithm on the benchmark instances presented by Montoya *et al.* (2017), and currently have many of the best known solutions.

Lee (2021) presents a Branch-and-Price method for the E-VRP-NL. However, in their work the recharging function is not linearized. This leads to a very complex pricing problem. The columns generated with the pricing problem represent *no-charge-segments*. No-charge segments are sequences of nodes, where the first and last nodes are either the depot or recharging nodes and the remaining are customer nodes. The master problem is an arc flow formulation using no-charge-segments as arcs.

2 Solution method

The solution method used is based on BP&C, which is arguably the most successful method for solving most extensions of the vehicle routing problem. For recent advances in BP&C for VRPs, see Costa *et al.* (2019). The master problem is a set partitioning problem and the pricing problem is a shortest path problem with resource constraints (SPPRC). The advantage of using a set partitioning formulation contrary to an arc flow formulation, as done by Lee (2021), is that the master problem formulation gives a tighter dual bound. In addition, in this case it results in the need to solve a single pricing problem. Lee (2021) needs to solve a pricing problem for each pair of start node, recharging node and end node. Its advantage is that each pricing problem will be significantly faster than the one solved when using a set partitioning formulation.

The SPPRC is solved by dynamic programming with a labeling algorithm. An important factor for a successful labeling algorithm, is the ability to identify and prune labels that are provably not a part of the optimal solution. Such pruning is called dominance. Pareto frontiers representing the recharging potential for the labels are compared to conclude on dominance. This feature also enables partial dominance to increase the efficiency of the labeling algorithm. Then, a subset of the feasible extensions to the end node is pruned. Partial dominance can also lead to complete dominance when several labels together dominate the entire pareto frontier of the dominated label.

Several well known techniques are used to improve the overall efficiency of the algorithm. For more information on the techniques see the survey by Costa *et al.* (2019). To speed up the pricing of columns, a bidirectional labeling algorithm has been implemented. NG-path is used to strengthen the dominance of labels and improve the efficiency of the labeling algorithm. The drawback is a weakening of the master problem. Limited memory subset row cuts are used to strengthen the dual bound.

The branching strategy is to first branch on the number of routes used, then the number of visits to each recharging node and lastly on the flow on each edge.

3 Preliminary results

The computational tests were run on a Lenovo ThinkSystem SD530, running CentOS 7.9.2009, with 3.6GHz Intel Xeon Gold 6244 CPU and 384Gb RAM. The algorithm is implemented in C++ and compiled with GNU GCC 10.3.0. The linear models are solved using Gurobi 9.1. The upper limit on the computation time was set to three hours. That is the same time limit as being used when testing the MILPs by Froger *et al.* (2019).

Our BP&C method is tested on the Montoya benchmark instances (Montoya *et al.*, 2017). It solves all 20 10-customer instances with an average computation time of about 13 seconds, compared to an average of about seven minutes for the path-flow model by Froger *et al.* (2019). They solve 5 of the 20 20-customer instances while our method solves 16. In addition, the model is able to solve 8 of the 20 40-customer instances, whereas Froger *et al.* (2019) solve none. These results are preliminary as we are continuing to improve our BP&C algorithm. Updated results will be presented at the conference.

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