Stable allocations for collaborative choice-based pricing in transport markets

V. Lurkin[a,*], L. Schlicher[b]

[a] University of Lausanne, HEC Lausanne Faculty of Business and Economics, Lausanne, Switzerland
virginie.lurkin@unil.ch,
[*] Corresponding author

[b] Eindhoven University of Technology, Department of Industrial Engineering and Innovation Sciences, Eindhoven, The Netherlands
l.p.j.schlicher@tue.nl

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1 Introduction

Horizontal agreements may qualify for exemption if they create sufficient pro-consumer benefits that outweigh the anti-competitive effects (Article 101(3), TFEU). Besides, they should not eliminate the competition in the relevant market, implying that participants should have a small market share and their combined market share should not exceed a specified limit (e.g., 20% in the Netherlands). A recent example of such an exemption stems from the Webtaxi case in Luxembourg, where the competition authority allowed various taxi companies to use a pricing algorithm to determine their taxi prices. While it was acknowledged that the joint use of such an algorithm constituted a situation of price fixing (i.e, a price agreement between competitors), it was decided that the agreement could be exempted since they expected huge pro-consumer benefits, mainly less waiting time, and lower prices, as well as more rides for drivers (Bostoen (2018)). At the same time, the combined market shares of the taxi operators would remain far below the threshold set by the Luxembourg authorities.

Inspired by this joint profit (re)allocation problem, we study a setting in which a set of transport operators (e.g., micromobility startups) can collaborate and decide at what price to offer sustainable urban mobility solutions (e.g., electric scooters or bikes) to a pool of travelers. To better reflect the decisions of these travelers, we assume that they choose among the services offered according to a multinomial logit model, one of the most widely-used disaggregate demand model (Ben-Akiva & Bierlaire (2003)).

Our work fits within the literature on choice-based pricing models, i.e., studies that integrate customer's choice behavior within pricing problems. These models are mathematically complex since they are nonlinear and non-convex in prices. Several equilibrium studies have been published on choice-based pricing problem involving multiple competitive firms (see *e.g.*, Lin & Sibdari (2009), Levin *et al.* (2009), Morrow & Skerlos (2011), Bortolomiol *et al.* (2021)). However, as already mentioned, unlike these studies and inspired by horizontal agreement exemptions, we assume that the firms can collaborate and collectively decide at what price to offer their services. Cooperative game theory is then the most appropriate methodology to adopt to allocate the associated joint profit between the firms. Our paper is therefore enrolled in the line of this existing literature on cooperative games inspired by real-life settings in transport.

By considering a choice model on the demand side, our setting involves pricing decisions that better capture the supply-demand interactions between the operators objective of maximizing their expected revenue and the travelers objective of maximizing the expected utility (Sumida *et al.* (2019)). To be in line with the conditions associated with the horizontal agreement exemptions, we assume that the transport operators set their prices in such a way that the total joint profit is maximized and their total market share remains constant, and as such remains below the authorized limit. In the next section we present a cooperative game for this setting, the transport choice (TC) game, and introduce various intuitive allocation rules.

2 A Cooperative Transport Choice Game

We consider a setting in which a group of homogeneous travelers is buying mobility services from a set of $N \subseteq \mathbb{N}$ transport operators. Each operator $i \in N$ offers one micromobility service (e.g., a e-bike or a segway) against price $p_i \in \mathbb{R}_+$ and cost price $c_i \in \mathbb{R}_+$. The mobility choices of travelers are represented using the logit model and we can therefore define the market share of transport operator i as the share of travelers that opts for mobility service $i \in N$. Following the logit model, this is given by

$$\frac{e^{\alpha_i - \beta p_i}}{1 + \sum_{j \in N} e^{\alpha_j - \beta p_j}}. \tag{1}$$

where $\alpha_i \in \mathbb{R}_+$ is an alternative-specific constant and $\beta \in \mathbb{R}_+$ a price sensitivity parameter.

Given this market share, the profit of transport operator $i \in N$ is defined by:

$$(p_i - c_i) \cdot \frac{e^{\alpha_i - \beta p_i}}{1 + \sum_{j \in N} e^{\alpha_j - \beta p_j}}. \tag{2}$$

We summarize this setting by tuple $\theta = (N, p, c, \alpha, \beta)$ with N the set of transport operators, $p = (p_i)_{i \in N}$ the vector of prices, $c = (c_i)_{i \in N}$ the vector of cost prices, $\alpha = (\alpha_i)_{i \in N}$ the vector of alternative-specific constants, and β the price sensitivity parameter. We refer to θ as the transport choice situation and let Θ be the set of all possible transport choice situations.

An example Let $\theta \in \Theta$ with $N = \{1, 2, 3\}$, $p = (6, 8, 15)$, $c = (8, 4, 1)$, $\alpha = (1, 0.5, 1.5)$ and $\beta = 0.36$. The market shares and associated profits corresponding to the initial prices of the transport operators are presented in Table 1. ◇

i	1	2	3
Price i	6.0	8.0	15.0
market share i	0.220	0.065	0.014
profit i	-0.440	0.260	0.199

Table 1 – *Prices, market shares and profits of the transport operators of situation θ*

For each TC situation $\theta \in \Theta$, we now introduce a cooperative game (N, v^θ), where N represents the set of players (i.e., transport operators) and v^θ represents the characteristic value function. In this game, $v^\theta(M)$ reflects the joint profit coalition $M \subseteq N \backslash \{\emptyset\}$ can realize. This joint profit is obtained by taking into account that (i) the sum of the market shares of the players in M remains stable (i.e., the new vector of prices should be such that the sum of their market shares remains the same) and (ii) all players outside coalition M (i.e., players in $N \backslash M$) keep their initially set prices. So formally, for every TC situation $\theta \in \Theta$, the associated cooperative transport choice (TC) game (N, v^θ) is defined by

$$\begin{aligned} v^\theta(M) &= \max_{x \in \mathbb{R}^M} \sum_{i \in M} (x_i - c_i) \frac{e^{\alpha_i - \beta x_i}}{1 + \sum_{j \in M} e^{\alpha_j - \beta x_j} + \sum_{j \in N \setminus M} e^{\alpha_j - \beta p_j}} \\ s.t. \sum_{i \in M} \left(\frac{e^{\alpha_i - \beta p_i}}{1 + \sum_{j \in N} e^{\alpha_j - \beta p_j}} \right) &= \sum_{i \in M} \left(\frac{e^{\alpha_i - \beta x_i}}{1 + \sum_{j \in M} e^{\alpha_j - \beta x_j} + \sum_{j \in N \setminus M} e^{\alpha_j - \beta p_j}} \right) \end{aligned} \quad (3)$$

for all $M \subseteq N \backslash \{\emptyset\}$ and $v^\theta(\emptyset) = 0$.

For every TC situation $\theta \in \Theta$, we can show that the optimal joint profit of any coalition $M \subseteq N \backslash \{\emptyset\}$, for all $M \subseteq N \backslash \{\emptyset\}$, is given by

$$v^\theta(M) = \frac{D^M(p)}{\beta(D^N(p) + 1)} \ln\left(\frac{D^M(c)}{D^M(p)}\right)$$

where $D^M(x) = \sum_{i \in M} e^{\alpha_i - \beta x_i}$. The proof of this results consists of three steps. First we relate our original optimization problem to another optimization problem that has a much simpler form of constraint. Then, we identify an optimal price vector and the associated optimal value for this new optimization problem, by using a Lagrangian type of optimality result from Bazaraa *et al.* (2013). Finally, we relate back these outcomes to our original problem.

In Table 2, you can see the optimal price vector and the corresponding market share and profit per transport operator for the **example** above.

i	1	2	3
optimal price i	13.980	9.980	6.980
market share i	0.012	0.032	0.255
profit i	0.074	0.190	1.523

Table 2 – *Prices, market share and profits of the transport operators of situation* θ

From Table 1 and Table 2, we learn that the joint profit, which is 1.787, exceeds the sum of individual profits without collaboration, namely $-0.440 + 0.260 + 0.199 = 0.019$. However, at the same time, we also observe that the individual profit of transport operator 2 decreases (from 0.260 to 0.190). So, in case of collaboration among the three transport operators, it would be natural that operator 1 and operator 3 would compensate operator 2 in some way.

The coalitional values of TC game (N, v^θ) corresponding to our **example** are represented in Table 3 below. You can see that the coalitional values of the individual coalitions (-0.440, 0.260 and 0.199) match with the profits of Table 1, and that the coalitional value of the grand coalition (1.787) matches with the sum of the profits of Table 2.

M	{∅}	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
$v^\theta(M)$	0	-0.440	0.260	0.199	0.230	1.485	0.756	1.787

Table 3 – *Coalitional values of game* (N, v^θ)

The next section shows how to distribute the overall joint profit amongst the different operators.

3 Allocation rule

It can be showed that some well-known allocation rules such as proportional rules and the Shapley value do not always belong to the core. One reason could be that these allocation rules do not explicitly compensate for the exchange of market share between players. Therefore, we present an allocation rule that does explicitly compensate for this exchange of market share. The allocation rule first allocates to each player the profit he/she generates under full collaboration, i.e., player $i \in N$ receives $(p_i^* - c_i)\frac{D^{\{i\}}(p^*)}{D^N(p^*)+1}$. Thereafter, we identify for each player $i \in N$ the increase (or decrease) in the market share, which is $\left(\frac{D^{\{i\}}(p^*)}{D^N(p^*)+1} - \frac{D^{\{i\}}(p)}{D^N(p)+1}\right)$. Player i then receives a price ϕ for each exchanged unit of market share, and pays the same price for each extra unit of market share. Formally, for each $\theta \in \Theta$ and associated TC game, the market share exchange (MSE) rule is given by

$$MSE_i = (p_i^* - c_i)\frac{D^{\{i\}}(p^*)}{D^N(p^*)+1} - \phi\left(\frac{D^{\{i\}}(p^*)}{D^N(p^*)+1} - \frac{D^{\{i\}}(p)}{D^N(p)+1}\right),$$

where the price ϕ is given by

$$\phi = \frac{v^\theta(N) - v^{\widehat{\theta}}(N)}{\left(\frac{D^N(p)}{D^N(p)+1}\right)},$$

with $\widehat{\theta} = (N, p, (p_i - 1/\beta)_{i\in N}, \alpha, \beta)$, i.e., $\widehat{\theta}$ is a TC situation with a constant marginal profit for all operators $(1/\beta)$ and with the same total market share as θ $(D(p)/(D(p)+1))$. Players cannot gain from such a TC situation (because $p_i - c_i = \frac{1}{\beta}$ for all $i, j \in N$). Hence, the numerator of ϕ represents the total additional return that is gained compared to a TC situation with the same total market share and where collaborating is not beneficial at all. This total additional return is then divided by the total market share. Indeed, ϕ can be recognized as the additional return per unit of market share. It can be showed that the allocations of the MSE rule always belong to the core. The proof of consists of two steps. First, we can show that the MSE satisfies efficiency, which follows by construction of MSE. Thereafter, we can show that MSE satisfies stability.

The allocations of the proportional allocation rules for (N, v^θ) are reported in Table 4. $\diamond$

		$\sum_{i\in M} x_i$	
M	$v^\theta(M)$	I-PROP	M-PROP
$\{1\}$	-0.440	-42.101	1.314
$\{2\}$	0.260	24.859	0.388
$\{3\}$	0.199	19.029	0.085
$\{1,2\}$	0.230	-17.242	1.702
$\{1,3\}$	1.485	-23.072	1.399
$\{2,3\}$	0.756	43.889	0.473
$\{1,2,3\}$	1.787	1.787	1.787

Table 4 – *Illustration of proportional allocation rules*

I-PROP is not in the core, since $\text{I-PROP}_1 + \text{I-PROP}_2 = -17.242 < 0.230 = v^\theta(\{1,2\})$. That means, players 1 and 2 together can earn more by breaking up and forming a new coalition together. Similarly, M-PROP is not in the core, since $\text{M-PROP}_3 = 0.085 < 0.199 = v^\theta(\{3\})$. That means, player 3 is better of by working individually. It can also be showed that the Shapley value, $SV = (0.407, 0.392, 0.989)$, does not belong to the core.

Unlike these rules, the allocation of the MSE rule that is given by $MSE = (0.738, 0.296, 0.753)$ does belong to the core.

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