# Alleviating string instability of adaptive cruise control using trajectory shaper

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# **1 INTRODUCTION**

Adaptive cruise control (ACC) systems are now widely equipped on recent commercial vehicles around the world due to the rapid advancement in vehicular automation technologies. As a critical component of Advanced Driver Assistance Systems (ADAS), the ACC system can alleviate driver's fatigue, and enhance safety and comfort during driving. However, recent studies indicate that ACC systems available on commercial vehicles are string unstable (Li et al., 2021), implying that vehicles driven by ACC systems will amplify the speed fluctuations coming from downstream traffic, thereby inducing more traffic oscillations.

The most direct and intuitive approach for guaranteeing string stability is to modify the ACC system (i.e., tuning controller parameters, or even revise the ACC algorithms). Many studies have investigated motion planning to achieve string stability. For example, the proportional-derivative (PD) and model predictive controller (MPC) and relevant variants have been studied extensively (Wang et al., 2019; Zhou et al., 2020). However, the nonlinearities and uncertainties in the vehicle dynamics lead to the actual vehicle trajectories deviating from the planned ones. This causes significant difficulties for analyzing and achieving string stability using an analytical approach. Moreover, as the autonomous vehicle industry is shifting from the rule-based control method (determine control decisions with explicit closed-form expression) to the deep neural network (DNN)-based end-to-end control method (Zhou et al., 2021), the complexity and intractability of DNNs used in deep reinforcement learning and imitation learning substantially increase the effort and cost to tune and revise the control algorithm of an ACC system. Therefore, directly tuning and revising an existing ACC system to achieve string stability can be rather expensive and laborious.

To mitigate the burden of revising an existing (string unstable) ACC system for achieving string stability, this study proposes a trajectory shaper-based method, which only modifies the trajectory information of the predecessor vehicle (before it is used by the ACC system), as shown in Figure 1. The proposed trajectory shaper is based on the notion that the trajectory information (i.e., position, speed) of the predecessor vehicle can have significant impact on the trajectory of the ego vehicle controlled by the ACC system. The design of the trajectory shaper is inspired by the input shaping approach (Singh and Singhose, 2002; Singhose et al., 1996) applied to attenuate residue vibrations

of mechanical systems (e.g., cranes, robot manipulators). Specifically, the trajectory shaper functions like a signal filter which removes undesired characteristics in the predecessor vehicle trajectory that will induce string unstable response from an ACC system. The merits of trajectory shaper are its simple formulation and straightforward implementation, which avoids modifying an existing ACC system. To ensure desired string stability using the trajectory shaper, only two parameters need to be identified from the historical trajectories: (i) the natural frequency, and (ii) the damping ratio of an ACC system. The natural frequency corresponds to oscillatory patterns in vehicle speed (which is related to the string instability). The damping ratio indicates how an ACC system alleviates the oscillatory speed patterns to achieve string stability. These two parameters provide the string stability characteristics for an existing ACC system. If these two parameters are accurately identified, a vanilla trajectory shaper (VTS) can be devised to alleviate string unstable response. However, the estimated natural frequency and damping ratio may not be accurate because the trajectory data from real-world measurements may be noisy (due to the measurement noise of onboard sensors) and may not be informative enough to retrofit the string unstable property of an ACC system explicitly. Thus, we include extra robustness (i.e., the capability to handle uncertain parameters in a model) into the trajectory shaper design to formulate a robust trajectory shaper (RTS) through a nonlinear program. The RTS can tackle different oscillatory speed patterns covering a span of natural frequencies and damping ratios to achieve string stability.



Figure 1 – Control setup of ego vehicle

## 2 METHODOLOGY

### 2.1 Vanilla Trajectory Shaper

The design of VTS is based on the method of zero vibration shaper (ZVS) which is designed to attenuate the residual vibration of an under-damped mechanical vibration system (Haidekker, 2020). The core idea of ZVS is to appropriately design two sequential impulses to negate residual vibrations, as shown in Figure 2(a). The first impulse  $A_1$  induces a vibrating response (blue curves), while the second impulse  $A_2$  induces another vibrating response (red curves). By sequentially applying impulses  $A_1$  and  $A_2$  to the under-damped mechanical vibration system, the vibrations induced from two impulses cancel out each other to achieve a vibration-free response. The residual vibration is analogous to the speed fluctuations of an ACC system (as both the mechanical vibration system and the ACC system can be described using the second-order dynamical system model); thereby, the core idea of VTS is the same as that of ZVS. Specifically, the VTS applies impulses  $A_1$ and  $A_2$  to convolute with the speed information of predecessor vehicle so that the speed variation information of the predecessor vehicle is appropriately altered (see the green dashed line in Figure 2(b)). The first impulse brings the speed information to an intermediate setpoint for the ego vehicle to respond. Next, when the ego vehicle is about to overshoot, the second impulse drives the speed information to the original speed level, which ensures that the ego vehicle converges to the speed of predecessor vehicle without overshoot (comparing the blue and green solid curves in Figure 2(b)). The performance index of ZVS/VTS is the residual vibration percentage induced by impulses:

 $V(\omega_0,\zeta) = e^{-\omega_0\zeta t_n} \sqrt{S(\omega_0,\zeta)^2 + C(\omega_0,\zeta)^2}$ (1) where  $\omega_0$  and  $\zeta$  are the natural frequency and damping ratio, respectively.  $S(\omega_0,\zeta) = \sum_{i=1}^n A_i e^{\omega_0\zeta t_i} \sin\left(\omega_0\sqrt{1-\zeta^2}t_i\right), C(\omega_0,\zeta) = \sum_{i=1}^n A_i e^{\omega_0\zeta t_i} \cos\left(\omega_0\sqrt{1-\zeta^2}t_i\right). A_i$  and  $t_i$  are the magnitude and time location (i.e., the time when an impulse is applied) of *i*th impulse, respectively. Variable  $t_1$  is the time when ZVS is initialized (e.g.,  $t_1 = 0$ ). Eq. (1) measures the vibrations (fluctuations) after applying *n* impulses to the under-damped second-order dynamical system. Correspondingly, ZVS aims to apply two sequential impulses (n = 2) to eliminate the vibrations (i.e., solving  $V(\omega_0, \zeta) = 0$  to obtain  $A_1, A_2, t_1$ , and  $t_2$ ).



(a) Core idea of ZVS

(b) Core idea of VTS

#### Figure 2 - Core ideas of ZVS and VTS

After solving  $V(\omega_0, \zeta) = 0$ , we obtain the magnitudes and times of the impulse sequence:  $A_1 = \exp(\zeta \pi/\sqrt{1-\zeta^2})/\left[1+\exp(\zeta \pi/\sqrt{1-\zeta^2})\right]$ ,  $A_2 = 1-A_1$ , and  $t_2 = t_1 + \pi/(\omega_0\sqrt{1-\zeta^2})$ . With the same idea and procedures, the VTS applies two sequential impulses to modify the original trajectory information of predecessor vehicle, such that the modified trajectory induces no speed overshoot/undershoot from the ego vehicle, which ensures its string stability. Correspondingly, by convoluting the impulse sequence with the original position  $p_{\text{pred}}(t)$  and speed  $v_{\text{pred}}(t)$  of the predecessor vehicle, the shaped speed and position in VTS are expressed as:

$$p_{\text{pred}}^{\text{shaped}}(t) = \sum_{i=1}^{n} A_i \, p_{\text{pred}}(t-t_i) \tag{2}$$

$$v_{\text{pred}}^{\text{shaped}}(t) = \sum_{i=1}^{n} A_i v_{\text{pred}}(t-t_i)$$
(3)

 $p_{\text{pred}}^{\text{shaped}}(t)$  and  $v_{\text{pred}}^{\text{shaped}}(t)$  will then be used as the position and speed information in the ACC system. Note that the delay effect is brought by the convolution, which ensures that the ego vehicle will converge to the desired speed and spacing slightly slower than following the original trajectory.

### 2.2 Robust Trajectory Shaper

In real-world applications, identifying the natural frequency and damping ratio accurately can be difficult, due to the noisy or even inaccurately measured trajectory information. The VTS implemented using inaccurate parameters will generate erroneous shaped trajectories, which degrades string stability performance. To avoid this, we incorporate robustness into the trajectory shaping design. Specifically, instead of enforcing the residual vibration percentage at a specific natural frequency and a specific damping ratio to be zero, we apply the idea of a specified input shaper (Singh and Singhose, 2002) so that residual vibration percentage is less than a tolerance level  $V_{\text{tol}}$  (e.g.,  $V_{\text{tol}} = 0.01$ ) over a span of natural frequencies and damping ratios. Then, defining the set of natural frequencies and damping ratios that can induce string unstable response as  $\Omega$  and  $\Psi$ , respectively, we apply the following nonlinear program to obtain the RTS (i.e., determining the magnitudes and time locations of impulses, and then convoluting with  $p_{\text{pred}}(t)$  and  $v_{\text{pred}}(t)$ ).

$$\min_{A_1,\dots,A_n,t_1,\dots,t_n} t_n \tag{4a}$$

s.t. 
$$t_{i+1} - t_i > 0$$
 (4b)

$$A_i > 0 \tag{4c}$$

$$\sum_{i=1}^{n} A_i = 1 \tag{4d}$$

$$V(\omega_0^j, \zeta^j) \le V_{\text{tol}}, \omega_0^j \in \Omega, \zeta^j \in \Psi$$
(4e)

The objective function in Equation (4a) aims to minimize the time location of the last impulse so that the delay effect introduced by convolution is minimized. Equation (4b) states that the time location of impulse occurring later is greater than the time location of earlier impulse. Equation (4c) constrains the magnitudes of impulses to be positive to reduce variations of the shaped trajectory. Equation (4d) ensures the shaped trajectory maintains the same magnitude as the original trajectory, so that the ACC system can still track the original trajectory. Equation (4e) bounds the residual vibration percentage below  $V_{tol}$  for all natural frequencies in set  $\Omega$ , and all damping ratios in set  $\Psi$ . The RTS can be efficiently solved using existing algorithms (e.g., interior-point algorithm).

## **3** INSIGHTS AND DISCUSSION

This study introduces two trajectory shapers that modify the trajectory information of the predecessor vehicle so that string stability can be maintained even if the ACC system is string unstable. The VTS is the most simple and straightforward one, which relies solely on accurately identified natural frequency and damping ratio to achieve desired string stability performance. The RTS uses nonlinear programming to consider a range of natural frequencies and damping ratios, which enhances robustness when dealing with ACC systems with inaccurately identified parameters.

Note that the historical trajectory of the ACC system plays an important role for both trajectory shapers, as it provides essential information for determining the natural frequency and damping ratio. If the historical trajectory is not sufficiently informative (e.g., containing only the free-flow speed, but not stop-and-go movements which reflect the string stability/instability property) to provide these two parameters, real-time iterative algorithms (e.g., extended Kalman filter, particle filters, etc.) are needed to continuously update these two parameters such that the trajectory shaper can achieve its desired performance. In addition, the acceleration/deceleration bound in the ACC system would also undermine the performance of the trajectory shaper, as the design of the trajectory shaper does not factor the saturation of acceleration/deceleration. To address this, a relaxation technique of acceleration/deceleration bound would need to be devised to augment the trajectory shaper.

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