Solving a Simultaneous Behavioral Decision Problem During Interactions Using Quantum Optimization

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1 PROBLEM STATEMENT

For smooth control of an autonomous vehicle, it is necessary to select acceleration/deceleration/lanes while considering the interactions with other vehicles. In particular, during merging at a ramp, vehicles that affect an agent are also affected by other vehicles. The propagation of these vehicle-vehicle interactions is illustrated in Figure 1. However, simulating the simultaneous selections made by vehicles during interactions is challenging.

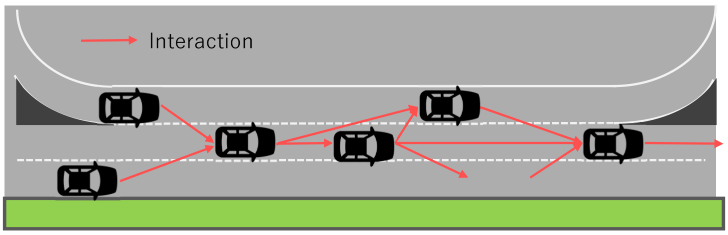

Figure 1 – *Example: Interaction between vehicles*

To solve the problem of simultaneous selection, we must determine the most likely probabilities of all interacting agents from all possible combinations of their choices:

$$\hat{\mathbf{a}} = \arg\max_{\forall \mathbf{z}} P(\mathbf{z}), \tag{1}$$

where $\hat{\mathbf{a}}, \mathbf{z}$ represent the combination of behavioral choices for all the agents, and P is the simultaneous probability of the behavioral combination. The number of possible combinations $\mathbf{a}$ is the exponent of the number of alternatives to the number of agents: $N_{alternative}^{m_{agent}}$. Hence, the optimization problem is NP-hard.

In this study, we solve this problem using quantum computing. For this, we formulate the simultaneous probability during interactions in such a manner that a quantum optimization technique can be applied. The optimization approach, which can help solve this problem quickly, enables us to obtain the most likely behavior choice for all the agents. A recently developed quantum computation approach was based on the Ising model. This approach can be reformulated as a quadratic unconstrained binary optimization (QUBO) with the least possible constraints.

Our formulations, based on a traditional social interaction model (Brock & Durlauf (2001)), can be transformed to a QUBO, and consequently, we propose a method to simulate simultaneous selections when the agents interact with each other.

Some existing activity simulators, for example, CEMDAP (Bhat *et al.* (2013)), evaluate interactions amongst a family or a small group. To the best of our knowledge, a longer chain of interactions has not been evaluated in previous studies. These chains of interactions can be observed in the vehicle movement on a ramp, two-dimensional choice of pedestrians in public spaces, and participation decisions during car-sharing, among others. The proposed method makes it possible to compute many similar phenomena involving social interactions.

2 FORMULATION

Brock & Durlauf (2001) proposed a local interaction model, wherein social utility S is given by

$$S(a_i, \mathbf{a}) = -E\left(\sum_{j \in c_i} J_{ij}(a_i - a_j)^2\right) \tag{2}$$

where a is a behavioral choice, c_i represents agents interacting with the decision-maker i, and J_{ij} is the weight of the interaction between agents i and j. When the social utility S is added to the observed utility u of the logit model, the probability of the choice is expressed as

$$P_i(a_i) = \frac{\exp\left(u(a_i) + S(a_i, \mathbf{a})\right)}{\exp\left(u(a_i) + S(a_i, \mathbf{a})\right) + \exp\left(u(a_i') + S(a_i', \mathbf{a})\right)} \tag{3}$$

where $a_i' \neq a_i$. In this study, we defined our problem as a binary choice problem. Equivalently, the numerator of Eq. (3) can be changed to

$$\prod_{j \in c_i} \exp\left(\frac{u(a_i)}{|c_i|} - \frac{J_{ij}}{|c_i|}(a_i - a_j)^2\right) \tag{4}$$

When a potential function $W_{ij}(a_i, a_j)$ is defined as

$$W_{ij}(a_i, a_j) = \exp\left(\frac{u(a_i) - J_{ij}(a_i - a_j)^2}{|c_i|}\right) \times \exp\left(\frac{u(a_j) - J_{ji}(a_i - a_j)^2}{|c_j|}\right), \tag{5}$$

the simultaneous probability $P(\mathbf{a})$ is

$$P(\mathbf{a}) = \prod_{\forall i} P_i(a_i) = \frac{\prod_{ij \in B} W_{ij}(a_i, a_j)}{\sum_{\forall \mathbf{a}} \prod_{ij \in B} W_{ij}(a_i, a_j)}, \tag{6}$$

where B is a set of interactions among agents, and the denominator represents a normalizing constant. The maximization of $P(\mathbf{a})$ can be transformed to a QUBO formulation, as follows:

$$\begin{aligned} &\max_{\mathbf{a}} P(\mathbf{a}) \Rightarrow \max_{\mathbf{a}} \prod_{ij \in B} W_{ij}(a_i, a_j) \\ &\Rightarrow \max_{\mathbf{a}} \sum_{ij \in B} \log\left(W_{ij}(a_i, a_j)\right) = \max_{\mathbf{a}} \sum_{ij \in B} \left(\frac{u(a_i) - J_{ij}(a_i - a_j)^2}{|c_i|} + \frac{u(a_j) - J_{ji}(a_i - a_j)^2}{|c_j|}\right) \end{aligned} \tag{7}$$

On representing the observed utility u as

$$u(a_i) = a_i u_{i1} + (1 - a_i)u_{i0} = a_i^2 u_{i1} + (1 - a_i^2)u_{i0} = (u_{i1} - u_{i0})a_i^2 + u_{i0}, \tag{8}$$

where $u(a_i = 0) = u_{i0}$ and $u(a_i = 1) = u_{i1}$, Eq. (7) becomes

$$\max_{\mathbf{a}} P(\mathbf{a}) \Rightarrow \max_{\mathbf{a}} \sum_{ij \in B} \left(\left(\frac{u_i - J_{ij}}{|c_i|} - \frac{J_{ji}}{|c_j|} \right) a_i^2 + \left(\frac{2J_{ij}}{|c_i|} + \frac{2J_{ji}}{|c_j|} \right) a_i a_j + \left(\frac{u_j - J_{ji}}{|c_j|} - \frac{J_{ij}}{|c_i|} \right) a_j^2 \right), \tag{9}$$

where $u_i = u_{i1} - u_{i0}$. Eq. (9) is in the form of a QUBO, and this enables us to obtain a solution for the optimal choice combination $\hat{\mathbf{a}}$ using a heuristic quantum annealing algorithm.

In general, by formulating it as a QUBO, any problem can be solved using quantum computation. However, quantum annealing is a natural computing mechanism that calculates the optimal solution using the unrestrained quantum behavior . Therefore, to efficiently solve these problems, it is desirable to include as few constraints as possible. The problem addressed in this study is a problem with no constraints; hence, it is a more suitable model to be solved using quantum computation.

3 NUMERICAL EXAMPLE

The numerical example presented herein verifies that our formulation for the behavioral decision problem during interactions can be solved using quantum optimization. The interaction network is considered as a clique network where the utility u_i and weight J_{ij} are randomly chosen in the intervals [-1, 1] and [0, 1], respectively. Considering a binary behavioral choice, simulations were conducted for different numbers of agents (5, 10, 20, 40, and 100). We used D-wave Leap, a real-time Quantum Application Environment, to solve the numerical examples. The quantum processing unit (QPU) used was an Advantage system 1.1 with 5436 working qubits. We also used the Python packages "dwave.system" and "dimod." For the simulations with 5, 10, and 20 agents, we verified that the optimal combinations obtained via quantum annealing are consistent with the exact solutions obtained by the enumeration of all possible combinations. This verifies the applicability of quantum optimization for the solution of this type of problem. Cases involving more than 20 agents cannot be solved by the enumeration method. Table 1 shows the computation time with respect to the number of agents. Despite the power-law scaling in the number of possible combinations ($2^{m_{agent}}$), owing to the use of quantum annealing, the difference in computation time is small, even when the number of agents increases. The number of samples in Table 1 refers to the number of trials required to solve the problem. The quantum annealing results are dependent on the initial setting, and hence, several trials are performed before the selection of the optimal one. Table 2 indicates that the computation time depends on the number of samples. Figure 2 depicts the distribution of the objective function (energy) and illustrates that the optimal result can be obtained even if the number of trials is small.

Table 1 – *QPU computation time based on number of agents*

# of agents	time (ms)	# of samples
5	8.41	100
10	8.55	100
20	9.45	100
40	12.85	100
100	14.25	100

Table 2 – *QPU computation time based on number of samples*

# of agents	time (ms)	# of samples
40	5.07	50
40	12.85	100
40	25.43	200
40	51.24	400

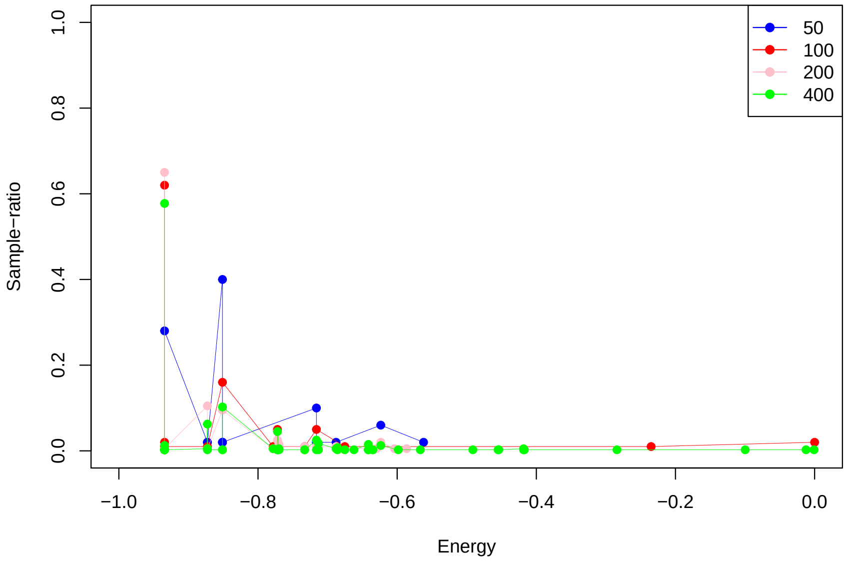

Figure 2 – *Energy distribution with respect to number of samples*

4 CONCLUSION

This paper proposed a method to determine the optimal selection of multi-agent behavior during social interactions by using quantum computation. The combination of actions with the maximum likelihood in a stochastic action selection problem is dependent on the probability maximization of the simultaneous probability, because of an interaction chain. However, this problem is NP-hard to compute. This study transformed the simultaneous selection probability during interactions to a QUBO that can be solved using quantum computation. This makes it possible to calculate the combination of behaviors that maximize the simultaneous selection probability. Our numerical calculations demonstrate that an exact solution can be obtained using quantum annealing. In addition, even in a situation where the number of agents is as large as 100 (the number of solution candidates is 2^{100}), it is possible to calculate the results quickly (15 ms) using the D-wave leap machine.

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