# Assessing Practical Railway Network Capacity using Petri Nets

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## 1 INTRODUCTION

Railway capacity represents one of the main characteristics of railway networks. It is important for understanding system limitations, current pricing and future investment decisions of the infrastructure manager, and consequently is an important factor for the current and future performance of this transport system. Railway capacity is typically measured by the absolute number of trains over a given time interval or the corresponding occupation times. It is impacted by factors such as the number of trains, the average train speed, the heterogeneity of train traffic and the timetable-induced time margins (s. UIC (2004)). Additionally, in complex networks we observe additional spatio-temporal dependencies due to heterogeneous rolling stock fleets, train services (routes, paths and frequencies) and service-related constraints (transfers and train orders).

Railway network capacity (RNC) can be defined as the maximum number of trains regarding the given service characteristics (i.e. timetable) and infrastructure. It is measured by the number of overall departing trains per time interval, regarding the resulting level-of-service (LOS) (i.e. average scheduled waiting time (SWT) per train). This is also known as practical RNC (s. Abril *et al.* (2008)). RNC is a recent research topic (s. Landex (2008), Jensen (2015), Rotoli *et al.* (2016), N. Besinovic (2017), D'Acierno *et al.* (2019), Kianinejadoshah & Ricci (2020) or Weik *et al.* (2020) as examples). However, the practical capacity i.e. determining maximal number of trains including the resulting LOS, has hardly been addressed in the network context.

This work aims for filling this gap by providing a new petri net (PN) approach for assessing the RNC, building upon the work of Burkolter (2005) and Farhi *et al.* (2017). Our main contributions are extending the existing PN approach from single railway lines to networks, adding an extension for assessing heterogeneous service frequencies, the identification of relevant capacity limiting (i.e. critical) processes and by providing a framework for RNC assessment.

## 2 METHODOLOGY

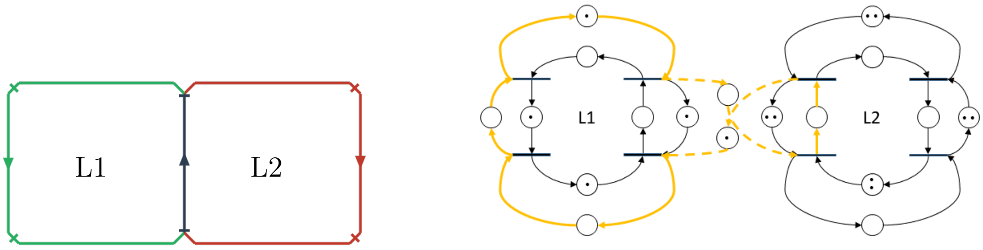
For assessing the practical RNC, a PN approach is developed. PNs are networks of places and transitions, which are a frequently used tool for modelling and visualization of complex processes (s. Petri (1962)). Being already well researched, PNs provide plenty of theoretical features, which have already been used for capacity assessment in the railway context (i.e. Olsder & Subiono

(1998) or Burkolter (2005)). Among the first, Farhi *et al.* (2017) applied the PN paradigm for modelling the traffic dynamics on a single metro line. The extensions to this approach, allowing for network wide capacity assessment, are shown in the following subsection.

## 2.1 Railway Network Occupation Model

The model takes as input and fixed: the infrastructure network, the service/line network, the local routing in stations and the line order/hierarchy. Furthermore, homogeneous rolling stock characteristics and driving dynamics are also assumed.

The modelling process starts by individually modelling the lines of the given service network as p-timed PNs, where the traversed sequence of infrastructure elements is captured. Assuming fixed-block operations, the lines are modelled with the granularity of individual blocks, each represented by two corresponding PN places, a running and a headway one. The associated timings represent the minimal running- and headway times. Introducing pairwise connection arcs, the networks representation is then obtained by connecting the line representations at sections of shared infrastructure (s. fig. 1b). Trains are modelled by putting token on the running-time place of a block. Heterogeneous numbers of trains per line are incorporated by using multiple token per train. The number of token per train for each line are thus set according to the least common multiple (lcm) over all numbers of trains per line in the network. The obtained network model is referred to as the railway network occupation model (RNOM). The relevant critical circuits of the RNOM are then identified by assessing the actual occupation (i.e. token) of the network. The headway (i.e. cycle mean) of the most limiting critical circuit (i.e. the one with the lowest headway) provides the capacity-limiting bound of network performance.



(a) *Railway Network Topology*

(b) *PN Graph Representation of the Topology (p-timings omitted for clarity)*

Figure 1 – *Example network of two symmetric lines L1 and L2*

Figure 1 provides an example of a railway network. Two lines, L1 and L2, consisting of 4 blocks each, are connected by two connection arcs (dashed arrows in fig. 1b) at one common block section (blue section in fig. 1a). In the PN, a train is represented by an occupied place on the inner loops. Empty blocks are indicated by the corresponding occupation of places on the outer loops. The differing numbers of trains per line and the resulting heterogeneous service frequencies are represented by multiple token. An exemplary critical circuit is indicated in yellow.

## 2.2 Railway Network Capacity Framework

The RNC framework is proposed for determining the capacity of the network. It applies the RNOM for assessing different occupation scenarios. A scenario consists of a certain number of trains per line, including their distribution in the network. Since capacity equals the maximum possible occupation, the scenario providing the highest headway on its relevant critical circuit is wanted. It is identified by enumerating over all possible scenarios between 1 train per line and the maximum number of trains in the network (at most 1 train per block). The framework is presented in fig. 2.

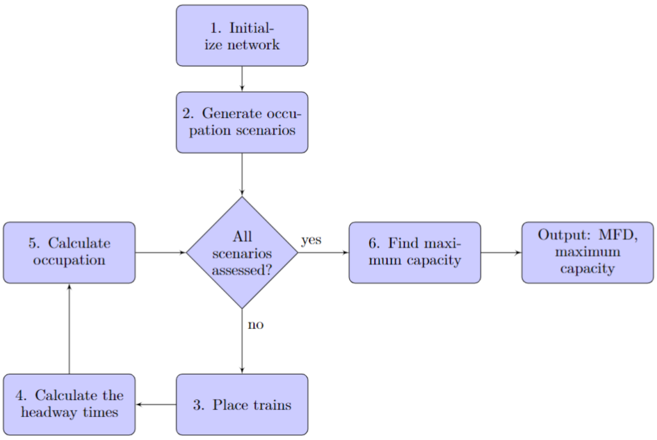
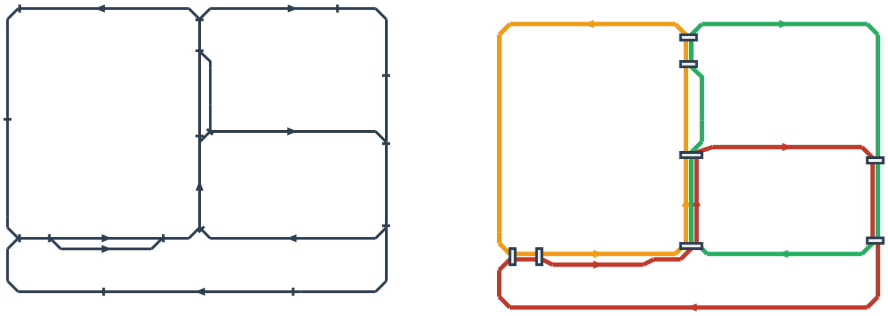


Figure 2 – *Railway network capacity framework*

In step 1, the RNOM is initialized as described in section 2.1. In step 2, the occupation scenarios to be assessed are generated. Ensuring to find the scenario representing the capacity (i.e. maximum occupation) of the network, one scenario is made for each combination of numbers of trains per line. The RNOM is then applied by setting the tokens according to the current scenario (step 3), identifying the relevant critical circuits (step 4) and calculating the corresponding occupation, relying on the headway of the identified circuit (step 5). Enumerating over all scenarios, the capacity is found by identifying the scenario with the maximum occupation (step 6). The frameworks provides the obtained maximum capacity and the corresponding occupation and LOS, visualized in a combined macroscopic fundamental diagram (MFD) (s. fig. 4).

## 3 RESULTS AND DISCUSSION

The presented RNC framework is used for assessing the capacity of an example network presented in fig. 3. The network consists of fixed-block infrastructure with homogeneous running and headway times (s. fig. 3a). A service network of three lines is operated, overall sharing 4 blocks in different parts of the network (s. fig. 3b).



(a) *Infrastructure of the Example Network (incl. block-markings and directions)*

(b) *Lineplan of the Example Network*

Figure 3 – *Example network with 3 lines (red, green and yellow) and 4 shared blocks*

The results of the capacity assessment are shown in an MFD (s.fig. 4), presenting the absolute number of trains (representing the actual occupation) and the resulting maximum train flows of all evaluated scenarios. Additionally, the avg. SWTs per train and scenario are given, representing the corresponding LOS. The RNC framework determines a network capacity of 10 trains/h for a range between 6 and 15 trains in total, proving the maximum flow is obtainable for multiple occupation scenarios. However, the most efficient maximum flow (i.e. with the least number of trains and the lowest avg. SWTs) can be reached for 6 trains only. As the maximum flow represents the aggregated line flows, it can be observed on the shared infrastructure section only.

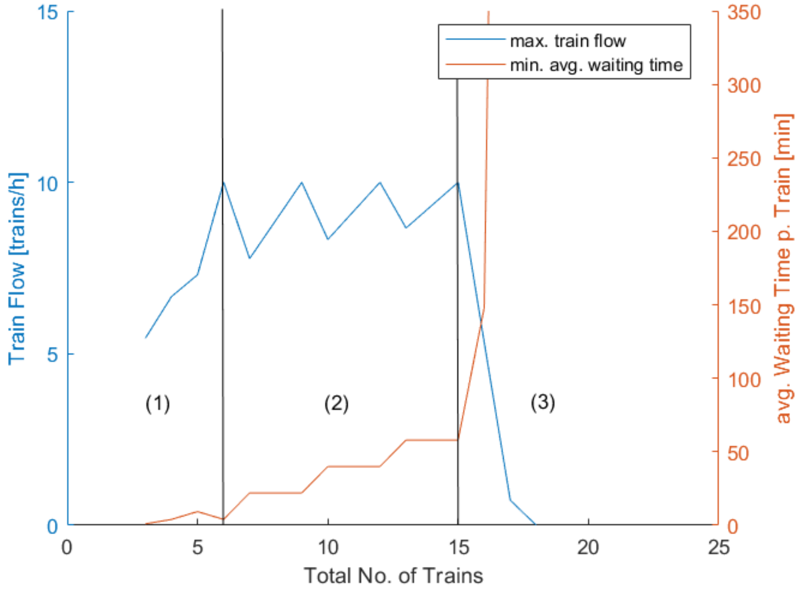


Figure 4 – *Combined MFD for the example network with the 3 traffic phases*

The MFD shows the existence of three distinct traffic phases: the phases of (1) free-flow, (2) maximum flow and (3) congested flow. The first phase is characterized by a steadily increasing flow with an increasing number of trains, since the network is empty and no train interactions cause waiting. Thus, the avg. SWTs are close to zero. Phase (2) is characterized by an almost constant[1] train flow for increasing numbers of trains. Caused by starting train interactions, increasing numbers of trains do not lead to increasing flow values but linearly increasing avg. SWTs. Thus, the flow-increasing potential of additional trains is getting compensated by the resulting increasing train interactions and the resulting hindrances. Phase (3) shows decreasing flows with increasing train numbers, caused by the saturation of the network. Thus, the occurring train interactions cause hyperbolic increasing SWTs and thus lead to a strong flow decrease.

The experimental results show that the traffic phases of the well known fundamental diagram (FD) of traffic flow also emerge in complex railway networks. Furthermore, the results highlight the relations between traffic flow and avg. SWTs, where each phase corresponds to a characteristic behaviour of the SWT. Also, the shape of the SWT is in line with findings of other capacitated systems i.e. queuing systems. Finally, the results show that the maximum flow can be reached with different occupation scenarios, whereas the most efficient maximum flow can be only reached for specific ones.

The obtained results provide an upper bound for the RNC. Testing the framework on different networks and real-life instances is expected for future research. Also, for practical application, future work should incorporate the effects of variable train orders and heterogeneous rolling stock characteristics. Capturing additional characteristic railway features, the proposed framework

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[1] The saw-tooth-like pattern emerges from the discrete nature of trains and resulting combinations of numbers of trains (or frequencies) per line respectively. Since the waiting times become minimal for homogeneous frequencies only, the corresponding distinct maximum points occur.

could then be used for supporting strategic, capacity efficient planning and thus might support the recent ambitions for efficient and eco-friendly transportation.

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