Assortment Optimization for Boundedly Rational Customers

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1 INTRODUCTION

In the last decade, app-based mobility services have received considerable attention as an interface between the service provider and the customer. The foundation of the app-based system is based on the real-time interaction between the booking system and the user's choice. Upon customers' arrival, a request is submitted to the platform. The platform then offers a travel menu (i.e., assortment) to the user to choose from. Inferring customer preferences and responding accordingly plays a vital role in app-based mobility services. One of the central decisions is which alternatives to include in the list of offered assortment to each arriving passenger.

Assortment optimization relies on choice models to estimate individuals' behavior. It is initiated by van Ryazin and Mahajan in their seminal paper which uses the multinomial logit (MNL) model for modeling customer behavior (Ryzin & Mahajan (1999)). It then developed for various choice models most of which assume that customers are rational agents. However, studies in marketing, economics, psychology (e.g., Simon (1957) and Hauser (1978)), and transportation (Di & Liu (2016)) have revealed that individuals' decisions may deviate from perfect rationality due to their cognitive limitations and biases. Such limitations and biases result in surprising outcomes which are known as boundedly rational/ irrational behaviors.

There are some cognitive assumptions that can account for humans' bounded rationality, such as reference-dependency of preferences, decoy effects, and lack of information or attention. The theory of reference dependency, pioneered by Tversky & Kahneman (1991), denotes that a reference point influences individuals' preferences between given alternatives. This phenomenon can explain cognitive biases in many choice situations, like transportation (Van de Kaa (2010)). Decoy effects, first studied by Huber *et al.* (1982), are choice reversals caused by the composition of choice sets. Such effects occur when adding a new alternative (i.e., decoy alternative) in a choice-set may increase the probability of choosing one of the former options (i.e., target alternative). Decoy effects have been acknowledged in various choice contexts (Rooderkerk *et al.* (2011)).

Despite the evolution of assortment optimization studies, bounded rationality is still largely overlooked in this research area. Most of the assortment optimization studies use the notion of consideration sets, proposed by Simon (1957), to capture humans' bounded rationality caused by their lack of information or attention. Reference-dependency of preferences and choice-set composition effects have been scarcely integrated into assortment planning frameworks.

This study incorporates reference-dependency of preferences and decoy effects in the assortment optimization model using the Random Regret Minimization model. We propose a greedy algorithm that is computationally tractable to find the optimal assortment when customers' behavior follows RRM. We have tested our algorithm for micromobility services. The results show that our proposed algorithm can find the optimal solution for all studied instances. Moreover, we compare the planned assortments against the widely used multinomial logit model to examine the effects of reference-dependency and choice set-dependency on the assortment decisions. Our results indicate that these behavioral phenomena have significant impacts on the optimal choice set, so they need to be taken into account by those who want to offer a menu of options to their customers.

2 METHODOLOGY

In this research, we employ the Generalized Random Regret Minimization (G-RRM) model proposed by Chorus (2014) to model customer choice behavior. We aim to select the optimal assortment of alternatives from the given universal set Ω including N alternatives so that the expected profit per customer is maximized. In our problem setting, alternative i is defined by the bundle of M attributes $(x_1^i, ..., x_M^i)$. The random regret of alternative i is composed out of a systematic regret R_i and an i.i.d. random error ϵ_i, $RR_i = R_i + \epsilon_i$. The systematic regret of alternative i is defined as the sum of the binary regrets that are associated with bilaterally comparing the attributes of alternative i with each of the other alternatives in the choice set. Thus, the systematic regret of alternative i in assortment s is written as below:

$$R_i(s) = \sum_{j \in s,\ j \neq i} \sum_{m=1}^{M} \left(\ln\left(\gamma + \exp\left[\beta_m.(x_m^j - x_m^i)\right]\right) - \ln(1+\gamma) \right) \tag{1}$$

Where β_m and γ denote the weight of attribute m and the regret weight, respectively. Also, the choice probability of alternative i is defined by Equation (2), where V_0 denotes the constant attraction value of the no-purchase option.

$$\pi_i(s) = \frac{\exp\left(-R_i(s)\right)}{V_0 + \sum_{j \in s} \exp\left(-R_j(s)\right)} \tag{2}$$

Let p_i denote the associated profit of alternative i. Therefore, the assortment optimization problem is formulated as follows.

$$\max_{s \subseteq \Omega} \sum_{i \in s} p_i.\pi_i(s) \tag{3}$$

The RRM model (1) is a reference-dependent choice model in which each offered alternative acts as a reference point for other options. It is also a convex function that is steeper in the loss than in the gain domain, thus (1) represents semi-compensatory and loss-aversion behavior. Loss aversion indicates that people are more sensitive to losses than gains, and semi-compensatory behavior suggests that the poor performance of an alternative in terms of some attributes can not be completely compensated with the good performance of other equally important attributes. Besides, the reference-dependent convex regret function enables the RRM model to predict decoy effects (See Chorus (2014) and Guevara & Fukushi (2016) for more details).

2.1 Solution method

Each optimization on Problem (3) could require the evaluation of all potential subsets of the universal set. Generally, assortment optimization is a combinatorial problem. It has been proven that the unconstrained assortment optimization under either MNL or NL choice model when dissimilarity parameters change in (0 1] (Davis *et al.* (2014)) can be solved in polynomial time.

Otherwise, the assortment optimization is an NP-hard problem. We propose a greedy algorithm to solve Problem (3). Algorithm 1 represents the pseudo-code of the proposed algorithm.

Algorithm 1 Greedy Algorithm for the Assortment Optimization Problem

```
Initialization: Set S := {s ⊆ Ω; |s| = 2},  s* := ∅,  O* := 0
for s ∈ S do
    O_s := Σ_{i∈s} p_i.π_i(s)
    while |s| ≤ N do
        k := argmax_{j∈Ω\s} Σ_{i∈s∪{j}} p_i.π_i(s)
        O_k := Σ_{i∈s∪{k}} p_i.π_i(s)
        if O_k ≥ O_s then
            s ← s ∪ {k}
            O_s ← O_k
        else
            break;
        end if
    end while
    if O_s ≥ O* then
        s* ← s
        O* ← O_s
    end if
end for
return s*
```

3 RESULTS AND DISCUSSION

The described methodology is coded with Python. As our running example, we consider an online vehicle-sharing service that provides two types of transportation services: (i) bike ($s = 1$) and (ii) scooter ($s = 2$). In this problem, alternative i is defined by three attributes: service type (s_i), the walking distance to the pick-up location (d_i), and associated price (p_i). We assume that there is a universal set including ten potential options as described in Table (1). The service provider offers each customer a set of alternatives so that the expected revenue per request is maximized.

Table 1 – *Universal set of potential alternatives.*

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Service type (s)	1	1	2	1	1	2	2	1	2	2
Walking distance (d)	100	110	120	130	135	140	145	150	155	300
Price (p)	25	24	25	18	17	17	16	12	19	20

To examine the proposed algorithm's performance and compare the RRM and MNL models, we generated multiple scenarios based on the parameters of the regret and utility functions in the RRM and MNL models. The regret weight (γ), weights of the service type (β_s), walking distance (β_d), and price (β_p) are defined in Table (3)). We employ Equation (1) for the RRM model. For the MNL model, we assume that the deterministic utility of alternative i is defined by $u_i = \beta_s.s_i + \beta_d.d_i + \beta_p.p_i$. We presume that the attraction value of the no-purchase option under both RRM and MNL models is 1.

Table (2) shows the optimal assortment under the RRM (S^*_{RRM}) and MNL (S^*_{MNL}) models as well as the proposed assortment by our algorithm (S^*_{Alg}). The expected revenue of the proposed assortments by the RRM (R^*_{RRM}), MNL (R^*_{MNL}), and algorithm (R^*_{Alg}) are also shown by this table. Under all scenarios, our proposed algorithm finds the optimal assortment, which is obtained by the complete enumeration.

Table 2 – *Optimal RRM and MNL assortments for different scenarios.*

No.	γ	β_s	β_d	β_p	S^*_{RRM}& S^*_{Alg}	R^*_{RRM} & R^*_{Alg}	S^*_{MNL}	R^*_{MNL}
1	0	0.5	-0.9	-2.9	{1,2,3,10}	18.67	{1,2,3,4,5,6,9,10}	16.13
1.1	0	10	-0.9	-2.9	{1,2,3,4,5}	24.98	{1,2,3,4,9,10}	17.04
1.2	0	0.5	-10	-2.9	{1,10}	24.93	{1,2,3,9,10}	18.56
1.3	0	0.5	-0.9	-10	{1,2,3,10}	18.86	{1,2,3,4,5,6,9,10}	16.08
2	0.5	0.5	-0.9	-2.9	{1,2,3,10}	18.61	{1,2,3,4,5,6,9,10}	16.13
2.1	0.5	10	-0.9	-2.9	{1,2,3,4,5}	23.51	{1,2,3,4,9,10}	17.04
2.2	0.5	0.5	-10	-2.9	{1,2,3,10}	21.36	{1,2,3,9,10}	18.56
2.3	0.5	0.5	-0.9	-10	{1,2,3}	18.46	{1,2,3,4,5,6,9,10}	16.08
3	1	0.5	-0.9	-2.9	{1,2,3,10}	18.62	{1,2,3,4,5,6,9,10}	16.13
3.1	1	10	-0.9	-2.9	{1,2,3,4,5}	21.40	{1,2,3,4,9,10}	17.04
3.2	1	0.5	-10	-2.9	{1,2,3,9,10}	20.43	{1,2,3,9,10}	18.56
3.3	1	0.5	-0.9	-10	{1,2,3}	18.43	{1,2,3,4,5,6,9,10}	16.08

We use this example to show that the RRM model can replicate the decoy effect, resulting from the reference- and choice set- dependency of preferences. To this end, we consider the optimal RRM assortment for Scenario 3.2 and calculate choice probabilities of the offered alternatives in the absence and presence of alternative 10. Let $s_1 = \{1, 2, 3, 9\}$ and $s_2 = \{1, 2, 3, 9, 10\}$, Table (3) denotes the choice probabilities of the offered alternatives for s_1 and s_2. Following the proposed approach by Guevara & Fukushi (2016), we define $\theta_i = \frac{\pi_i(s_2)}{\pi_i(s_1)}$ for alternative i to assess the impact of alternative 10 on market share of alternative i. If $\theta_i > 1$ then alternative 10 is a decoy for alternative i.

Table 3 – *Decoy effect in the optimal RRM assortment for Scenario 3.2*

	1	**2**	**3**	**9**	**10**
$\pi_i(s_1)$	0.2718	0.228	0.189	0.099	-
$\pi_i(s_2)$	0.272	0.239	0.209	0.141	0.0002
$\theta_i = \frac{\pi_i(s_2)}{\pi_i(s_1)}$	1.003	1.048	1.102	1.422	-

4 CONCLUSIONS

This study develops an assortment optimization framework that incorporates humans' bounded rationality. The proposed bounded rational framework leads to different assortment and purchase decisions made by the supply and demand sides, respectively. We explain these differences in light of the different behavioral assumptions of our model. We propose a greedy algorithm that can solve the problem. In the next phases of this research, we will apply the algorithm to bigger choice sets and elaborate on mathematical proofs.

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