A subsidy-stabilized assignment game for Mobility-as-a-Service markets with both fixed route and on-demand operators

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Extended abstract submitted for presentation at the 11th Triennial Symposium on Transportation Analysis conference (TRISTAN XI)
June 19-25, 2022, Mauritius Island

January 15, 2022

Keywords: Branch-and-bound, Mobility-as-a-Service, Mobility-on-demand, Assignment Game

1 INTRODUCTION

New mobility services are emerging rapidly in recent years, such as bikeshare, micromobility, carshare, ride-hail, and microtransit, which have given rise to the concept of Mobility-as-a-Service (MaaS). MaaS platforms provide mobility services through a joint digital channel that enables users to plan and pay for multiple types of mobility services. Services include both fixed route and mobility on-demand (MOD). There's a growing need to develop analytical tools to help design different MaaS platforms: for the platforms, which operators to include, what mechanisms to use to allocate costs; for operators, which service regions or routes to serve, what service capacities to operate, what prices to charge passengers. MaaS systems are two-sided markets, with two mutually exclusive sets of agents, i.e., users and operators. Assignment models in MaaS systems need to depend on both users' route choices in the multimodal service network context and operators' pricing and operation decisions.

A cooperative game-theoretic framework has been proposed to model the stable matching between operators and users in a two-sided market. Pantelidis et al. (2020) proposed a many-to-many assignment game to model MaaS systems, where each route can be operated by one or more operators and the solution is mechanism-agnostic (so long as a mechanism falls between a buyer-optimal or seller-optimal stable outcome space). The assignment game is a special case bilevel problem in which the lower level is the matching and the upper level is the cost allocation design; unlike conventional bilevel problems there is no dependency from the lower level on the upper level decisions except for solution feasibility (i.e. stable outcome space is non-empty). Existence of a feasible and stable outcome depends on the complicated relationship between trip utility, travel cost, operation cost, and ownership of different parts of the network. If some matching pairs do not provide enough gain to be allocated between users and operators, the users may deviate from the routes selected in optimal matching or just end up unserved. The solution provided in Pantelidis et al. (2020) is only to the lower-level problem: a matching solution may be found that does not exhibit any stable outcomes and is hence not the solution to the bilevel problem. The best stable assignment can be improved upon with subsidy to make an unstable solution stable. The MaaS platform can intervene by injecting subsidies to the infeasible/unstable matching pairs to increase their gain to make sure that no user deviates from the matching. Tafreshian and Masoud (2020) used a minimum subsidy problem to obtain a stable outcome for an infeasible/unstable matching in peer-to-peer ridesharing matching games, which showed that subsidization is an effective way of stabilizing matchings. The more generalized assignment game with subsidy-stabilization (without subsidy is a more constrained variant) is considered.

Secondly, the model from Pantelidis et al. (2020) only considers fixed route services. MaaS markets include many MOD services: microtransit, shared bikes, taxis, etc. These services cannot be modeled using service links and capacities, but instead need to consider service region design (which zones are served), fleet size, and the resulting matching frictions (e.g. Zhou et al., 2021). The matching subproblem from Pantelidis et al. (2020) needs to be changed from a conventional multicommodity network design problem to one that includes nonlinear matching friction functions as the continuous approximation of zonal performance functions for MOD service, resulting in a nonlinear integer programming problem within a special case bilevel optimization.

We propose a branch-and-bound based solution algorithm to find a subsidy-stabilized solution to this many-to-many assignment game that ensures stability for a MaaS market, considering both fixed-route services and MOD. This contribution significantly extends the model proposed by Pantelidis et al. (2020) making it applicable to a much wider set of MaaS networks and guaranteeing stability through platform intervention.

2 METHODOLOGY

Traditional fixed-route transit services and MOD services operate with different strategies but should be both considered in MaaS systems.

To model the 2 types of services, we create a joint service network by connecting MOD complete subgraphs with a fixed-route network with access and egress links (the latter of which is modeled in Pantelidis et al., 2020). Let (N, A) be the joint network, F be the set of all operators and F_{MOD} be the set of MOD operators. Travel cost of access links and operation costs of MOD operators are modeled with Cobb-Douglas production functions. Travel costs of access links are shown in Eq. (1), where $t_{ij,acc}$ is a user's cost to access link $(i, j) \in A$, h_f is the fleet size of MOD operator $f \in F_{MOD}$ that operates in a service region that includes node $j \in N$, x_{ij} is the flow of access link $(i, j) \in A$, and a_{acc}, b_1, b_2 are parameters ($b_1 < 0, b_2 > 0$) that can fit the function to specific operators that account for their matching algorithms, travel patterns, and built environment structure. Operation cost of MOD operator is shown in Eq. (2), where $c_{f,MOD}$ is the operation cost of MOD operator $f \in F_{MOD}$, N_f is the set of nodes operated by $f \in F_{MOD}$, , $\sum_{j\in N_f} x_{ij}$ is the sum of all the flows served by $f \in F_{MOD}$, a_f, b_3, b_4 are similarly calibrated parameters ($b_3, b_4 > 0$).

$$t_{ij,acc} = a_{acc} h_f^{b_1} x_{ij}^{b_2} \tag{1}$$

$$c_{f,MOD} = a_f h_f^{b_3} (\sum_{j\in N_f} x_{ij})^{b_4} \tag{2}$$

The objective function of the lower-level matching problem (L_1) is written as the sum of the users' travel cost, operators' operation cost (both fixed-route and MOD), where the operation costs of MOD include costs of operating fleets ($c_{f,MOD}$) and fixed infrastructure costs at the MOD nodes in their services. The MOD operators' choices (service region with given fleet size), fixed route operators' choices (which links to operate with given service capacity), and users' choices (multimodal service path flows) are solved from L_1. The upper level of the model is pricing and cost allocation (L_2). The formulation is similar to Pantelidis et al. (2020). Since the matching from L_1 is link-based, path-based flows are not unique, but the model is still meaningful due to the uniqueness of system-level measures such as total operation costs and network flows. If infeasibility or instability is encountered in L_2, subsidy stabilization is introduced. L_3 solves for minimum amount of subsidy to either users or operators subject to the cost allocation constraints, which is the minimum of subsidy needed to stabilize an unstable matching from L_1.

An original branch-and-bound algorithm is proposed to solve the assignment game with subsidy. $L1$ is a nonlinear integer program, which can be solved with the branch-and-bound algorithm (Land and Doig, 2010); at each branch, a convex nonlinear optimization is solved. The bound used to prune branches is the sum of system cost and subsidies, i.e., the subsidized system cost. When an integer solution is found, we solve the cost allocation problem ($L2$) for the corresponding matching. If a stable outcome is achieved, whose system cost is lower than lowest subsidized system cost found yet, the upper bound is updated. If infeasibility or instability is reached instead, we solve the minimum subsidy problem ($L3$) to find the subsidized system cost to see if the upper bound needs updating. All branches with an objective value larger than the upper bound of the subsidized system cost are pruned. When all branches are closed, the corresponding matching of the current upper bound (and corresponding stable outcome space) is output as the optimal solution of the bilevel problem with subsidy. The algorithm can be further extended into a branch-and-cut algorithm.

Analogies to system optimal (solution from Pantelidis et al., 2020), user equilibrium without pricing (optimal stable outcome without subsidy), and user equilibrium with optimal pricing (subsidy-stabilized optimum) solutions are presented, where the first best subsidy is not guaranteed to attain the system optimum.

3 NUMERICAL EXAMPLE

We use a toy network shown in Fig. 1 to illustrate how the method works. In Fig.1(a), the solid links represent the fixed-route services, the same color are operated by the same operator. There are 3 MOD operators (blue, green, brown). The circles represent the nodes that MOD operators can choose from (blue: A,B,C; green: B,C; brown: B,D). The network can be expanded into the network in Fig.1(b) by adding access link and egress link of MOD. Travel cost, operation cost, and capacities are labelled in Fig.1. The values of the parameters are set as follows: $a_{acc} = 1$, $b_1 = -2, b_2 = 1$, $a_f = 2$, $b_3 = 2, b_4 = 1$. The fleet sizes are given in Table 1. The infrastructure cost of MOD nodes 7, 8, 9, 10, 11, 12, and 13 are 3, 3, 0, 0, 1, 1, and 3, respectively. Demand is 1,000 from nodes 1 to 3, and 500 from nodes 1 to 4. The branch-and-bound algorithm found 18 integer solutions before closing all branches. Each branch is solved using Gurobi. The top 5 solutions are shown in Table 1.

4 COMPUTATIONAL EXPERIMENTS

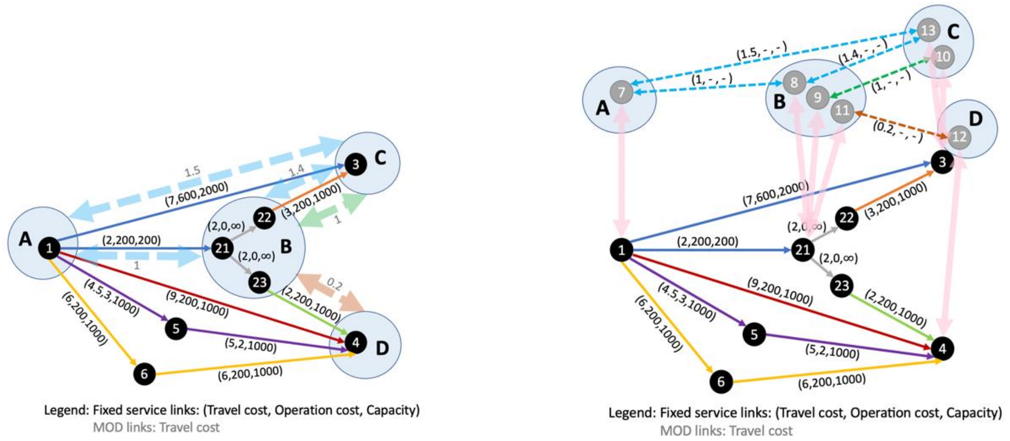
Several in-depth computational experiments will be presented. These include:

- Randomized instances of varying sizes to evaluate computational efficiency of solution algorithm
- A case study with a network instance drawn from a parametric city design (Fielbaum et al., 2017) in which three scenarios are investigated:
    - Only fixed route service provided (with two different service frequency options), under different utility levels to illustrate operators' incentive to enter to the platform
    - The same service network with added MOD operators (with two different fleet size levels) to illustrate the model's sensitivity to identify conditions where MOD operations

work better as first/last mile, and sensitivity of operators' revenues and ridership to fleet size and service frequencies

- Splitting of the user demand into two population segments to demonstrate the effect of user heterogeneity and assessment of service equity

The algorithm is a powerful tool of evaluating MaaS system performance, which can facilitate the decision-making and intervention of MaaS platforms and other regulators. The framework can be further applied to two-sided market use-cases under a network setting, such as freight logistics and airline revenue management problems.



(a) Original Network (b) Joint Network

Figure 1 – *Toy Network*

Table 1 – *Top 5 Solutions of the Toy Network*

	Fixed-route links operated	*MOD Nodes Opened*	*MOD Fleet Sizes (blue, green, brown)*	*System Cost*	*Subsidies needed*	*Subsidized System Cost*	*Found*
1	(23,4),(1,3),(1,5),(5,4),(1,21),(21,23)	7,11,12,13	0.96, 0.21, 0.87	11,854.63	2.50	11,857.13	1st
2	(23,4),(1,3),(1,5),(5,4),(1,21),(21,23)	7,13	0.96, 0, 0	11,854,71	2.50	11,857.21	2nd
3	(23,4),(1,3),(1,5),(5,4),(1,21),(21,23)	11,12	0, 0, 0.79	11,854.92	2.50	11,857.42	6th
4	(23,4),(1,3),(1,5),(5,4),(1,21),(21,23)	-	0, 0.11, 0	11,855.00	2.50	11,857.49	11th
5	(23,4),(1,3),(1,5),(5,4),(1,21),(21,23)	7,12,13	0.95, 0.35, 0	11,855.70	2.50	11,858.20	3rd

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