Strategic planning under uncertainty in transportation networks

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Logistics service providers (LSPs) utilize transportation assets such as car wagons and containers to satisfy customer demand. Servicing such demand requires that assets of appropriate capacity should be utilized for the transportation operation. The amount of time required for transportation operations can be affected by network disruptions and is therefore stochastic. This gives rise to a generalized newsvendor problem, where the LSP decides how many assets to acquire and how to schedule them so that stochastic demand requests that require specific capacity but uncertain time are fulfilled. Excess demand can be satisfied by utilizing spot assets, which, however, come at a higher cost. We apply this framework to an LSP who leases train wagons on an annual basis and responds to demand fluctuations via ad-hoc spot train rentals and formulate this problem as a two-stage stochastic program. Structural properties of the formulation and a column generation-based procedure greatly reduce the search space of optimal solutions. Computational results illustrate the efficiency of our approach.

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1 INTRODUCTION

Logistic service providers (LSPs) offer door-to-door transportation services to companies spanning across multiple industries. In order to carry out such services with high operational efficiency, their capability to respond to fluctuating customer demand is of paramount importance. Timely access to transportation infrastructure, such as train wagons and containers, requires the deployment of assets provisioned well in advance, so that resorting to reactive, last-minute alternatives is kept to a minimum.

Consider, for instance, a provider of rail transportation services, who may commit to the acquisition of a certain number of train wagons on an annual basis, either by entering in long-term leasing agreements or by purchasing the wagons herself. If her customers' demand exhibit strong seasonal fluctuations and she needs additional wagons to cover it, she will have to use spot wagons, which are often priced at a premium. Therefore, committing to a small number of wagons for long-term usage exposes her to demand variability, leading to last-minute orders and recurrent, one-off costs. Committing upfront, however, to an unnecessarily large number of wagons can lead to significant fixed costs, affect liquidity, and ultimately the prosperity of her operations. Such a position can lead to financial distress, especially if anticipated demand is not realized.

Although such decisions have additional complexities, in the case of transportation assets there is one aspect that makes them particularly challenging: customer demand is in the form of transportation services, while supply for those services is in the form of asset acquisitions. Moreover, requests for transportation services have two dimensions of uncertainty: the number of requests per customer, and the time spent for each request, which is the round-trip duration to the customer and the time spend in her premises to carry out the service. Translating such service requests to strategic asset acquisition decisions is challenging, and depends critically on the efficiency of the underlying operations. The literature has considered this problem in several specific settings (Yamada *et al.*, 2009, Nourinejad & Roorda, 2017, Noori-daryan *et al.*, 2017, Meng *et al.*, 2014), but, to the best of our knowledge, our work is the first one to provide a unifying, stylized framework that captures this trade-off.

In this work, we formulate this problem as a two-stage, multi-period stochastic mixed-integer program and solve it using column generation and a customized algorithm. We provide computational results that demonstrate the efficiency of our approach.

2 METHODOLOGY

We use the following notation for our model.

Sets.

$C = \{1, \ldots, |C|\}$, Customers, indexed by c.

$T = \{1, \ldots, |T|\}$, Time periods, indexed by t.

$N = \{1, \ldots, |N|\}$, Assets, indexed by n.

$S = \{1, \ldots, |S|\}$, Scenarios, indexed by s.

Parameters.

ρ, Maximum duration an asset can be in service per time period.

θ_{cts}, Service request demand of customer c in time period t under scenario s (number of trips required)

τ_{cts}, Duration an asset is deployed to perform one service request of customer c in time period t under scenario s

k_t, Number of spot assets available during time period t at the spot market.

p_{ts}, Probability that in time period t scenario s realizes.

Λ, Regular asset acquisition cost.

Γ, Spot asset acquisition cost.

Decision variables.

ω, Number of assets acquired at the beginning of the planning horizon (regular assets).

ζ_{ts}, Number of additional assets rented from the spot market, during time period t under scenario s (spot assets).

y_{nts}, $= 1$, if asset n is utilized in time period t under scenario s, 0 otherwise

A_{cnts}, Number of service requests of customer c satisfied by asset n, during time period t under scenario s.

Using the sets, parameters and variables, the Strategic Model (SM) is defined as follows.

$$\min \Lambda\omega + \Gamma \sum_{t\in T}\sum_{s\in S} p_{ts}\zeta_{ts} \tag{1}$$

$$\text{s.t. } \omega + \zeta_{ts} \geq \sum_{n\in N} y_{nts}, \qquad \forall t \in T,\ \forall s \in S \tag{2}$$

$$\sum_{n\in N} A_{cnts} = \theta_{cts}, \qquad \forall c \in C, \forall t \in T,\ \forall s \in S \tag{3}$$

$$\sum_{c\in C} \tau_{cts} A_{cnts} \leq \rho y_{nts}, \qquad \forall n \in N,\ \forall t \in T,\ \forall s \in S \tag{4}$$

$$0 \leq \zeta_{ts} \leq k_t; \quad \zeta_{ts} \text{ integer} \qquad \forall t \in T,\ \forall s \in S \tag{5}$$

$$y_{nts} \in \{0,1\}, \qquad \forall n \in N,\ \forall t \in T,\ \forall s \in S \tag{6}$$

$$A_{cnts} \geq 0; \quad A_{cnts} \text{ integer} \qquad \forall c \in C,\ \forall n \in N,\ \forall t \in T,\ \forall s \in S \tag{7}$$

$$\omega \geq 0; \quad \omega \text{ integer} \tag{8}$$

The objective function minimizes the total expected cost, which consists of the cost of the regular assets and the expected cost of spot assets. Constraints (2) state that the assets required should be lower than or equal to the total number of available assets, spot and regular, for every time period and scenario. Constraints (3) show that service requests from customer c, θ_{cts}, must be satisfied by asset deployments in a scenario/ time period combination. Constraints (4) pose that the total time an asset is in use during a time period under a scenario should be no higher than its available operating time in every scenario/ time period combination. Finally, Constraints (5)-(8) show the bounding and integrality restrictions of the decision variables.

A key property of (1)-(8) is that the variables that are in the objective function appear only in constraints (2), lower bounded by $\sum_{n \in N} y_{nts}$, the number of required assets in each period and scenario. Therefore, a simpler version of the SM model (1)-(8) can be formulated assuming that the number of required assets in each time period and scenario, δ_{ts}, is given exogenously. We use $\boldsymbol{\delta}$ to denote the vector of δ_{ts}. Then, model (9)-(12) describes this reformulation.

$$z(\boldsymbol{\delta}) = \min \ \Lambda\omega + \Gamma \sum_{t \in T} \sum_{s \in S} p_{ts}\zeta_{ts} \tag{9}$$

$$\text{s.t.} \ \omega + \zeta_{ts} \geq \delta_{ts}, \qquad \forall t \in T, \ \forall s \in S \tag{10}$$

$$\omega \geq 0, \qquad \omega \text{ integer} \tag{11}$$

$$\zeta_{ts} \in \{0, 1, \ldots, k_t\}, \qquad \forall t \in T, \ \forall s \in S \tag{12}$$

Here, the objective, (9), is identical to the original objective, and constraints (10) state that there need to be at least as many assets available in each time period/scenario combination as there is demand for. For model (9)-(12), we can formally prove the following result, which we state hereby as a remark.

Remark 2.1 *Define*

$$\delta^*_{ts} := \min \left\{ \sum_{n \in N} y_{nts} : (3) - (4); (6) - (7) \right\} \tag{13}$$

- *If* $\delta_{ts} = \delta^*_{ts}$ *for all* $t \in T, s \in S$, (9)-(12) *and* (1)-(8) *have the same optimal objective value.*
- *If* $\delta_{ts} \geq \delta^*_{ts}$ *for all* $t \in T, s \in S$, *then the optimal objective value of* (9)-(12) *is an upper bound to the optimal objective of* (1)-(8).
- *If* $\delta_{ts} \leq \delta^*_{ts}$ *for all* $t \in T, s \in S$ *with at least one inequality strict, then the optimal objective value of* (9)-(12) *is a lower bound to the objective solution of* (1)-(8).

Remark 2.1 can be used to devise an algorithm to solve (1)-(8): calculate δ^*_{ts} by solving (13) for each t, s, and then use them to solve (9)-(12). The former step can be done efficiently with a column generation algorithm, because (13) has a cutting-stock problem structure. In the original cutting-stock problem, the goal is to cut paper rolls in a number of pieces of a certain length (not necessary identical lengths) while minimizing the total number of paper rolls that are needed. In our setting, the problem is to select *schedules* defining the number of service requests an asset will satisfy of different customers during time period $t \in T$ under scenario $s \in S$, i.e. a cutting pattern represents the allocation of an asset to customers. The set of all possible asset schedules is denoted by Q. Then, the service requests satiated by an asset to customer $c \in C$ in asset schedule $q \in Q$ is signified by v_{qc}. We drop the indexes t and s for brevity and define the integer variable λ_q, which represents the number of assets following schedule q. Then, subproblem (13) can be formulated as follows.

$$\min \ \sum_{q \in Q} \lambda_q \tag{14}$$

$$\text{s.t.} \ \sum_{q \in Q} \lambda_q v_{qc} = \theta_c, \qquad \forall c \in C \tag{15}$$

$$\lambda_q \geq 0; \quad \lambda_q \in \mathbb{Z} \qquad \forall q \in Q \tag{16}$$

We then state a proposition that shows how using the knowledge of δ_{ts} we can calculate ω and ζ_{ts}.

Proposition 2.1 *Model (9)-(12) is convex and piecewise linear in ω. Further, let $\omega_{min} = \max_{t\in T}\{\max_{s\in S}\{\delta_{ts}\} - k_t\}$ and $P = \{(t,s) : \delta_{ts} \geq \omega_{min}\}$. Then, the optimal number of regular assets ω^* is in the set $\Omega^* := \{\omega_{min}\} \cup \{\delta_{ts}, \forall (t,s) \in P\}$ and the optimal number of spot assets follows from $\zeta^*_{ts} = \max(0, \delta_{ts} - \omega^*)$.*

3 RESULTS

In our algorithm, we use column generation to solve (13) and then a simple line search across all feasible δ^*_{ts} to find the optimal ω, as implied by Proposition 2.1. To test our approach, we have generated instances with varying periods ($|T| \in \{26, 62\}$), customers ($|C| \in \{2, 4, 8, 16\}$), scenarios $|S| \in \{5, 10, 20, 50\}$ and capacity tightness ($\frac{\sum_{c\in C}\theta_{cts}}{k_t} \in \{0.6, 1\}$). The combination of parameters leads to 192 instances, which were classified to small, medium and large according to

		Gap (%)		CPU (s)		No solution	
Problem size	Instances	CPLEX	CG	CPLEX	CG	CPLEX	CG
Small	54	**0**	**0**	**1.23**	15.86	**0**	**0**
Medium	90	1.26	**0.03**	3,978	**255**	7	**0**
Large	48	7.11	**0.01**	7,200 (TL)	**1,728**	35	**0**

Table 1 – *Average values over the Small, Medium, and Large instances for CPLEX and our custom algorithm (CG). TL denotes time limit.*

4 DISCUSSION

We study the strategic planning of transportation assets, a problem faced by LSPs, emphasizing that demand for services is expressed in the form of service requests and time during which the servicing asset is occupied. Our formulation captures the core elements of this problem, and as such it can be extended to more specific situations. We utilize a decomposition scheme where each subproblem, formulated as a cutting-stock problem, is solved by column generation and their solutions are utilized in a reduced reformulation of the original problem, which we can solve in polynomial time by exploiting its structure.

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