A demand-responsive bus system for peak hours with capacitated vehicles

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Extended abstract submitted for presentation at the 11th Triennial Symposium on Transportation Analysis conference (TRISTAN XI)
June 19-25, 2022, Mauritius Island

March 31, 2022

Keywords: demand responsive bus services, flexible bus services, mixed integer programming

1 INTRODUCTION

When demand for transportation is much higher in one direction than in the opposite direction, buses are often overcrowded in one direction and almost empty in the other. This happens mostly because of the lack of flexibility during operations of most traditional bus systems. To provide a better service, many demand-responsive systems have recently been studied. Vansteenwegen et al. (2022) provide a literature review on such systems.
To improve the efficiency of service in a corridor with uneven demand, Furth and Day (1985) suggest applying stop-skipping. The authors provide valuable input on the advantages and disadvantages of several strategies, however, the optimization of the systems are still lacking.
In this study, we focus on a single line where the demand for transportation in one direction during peak hours is much larger than the demand in the opposite direction and the demand outside peak hours. This imbalance in the demand is exploited to provide a better service for the larger flows. The system we propose aims to increase the frequency of a service towards a city center during morning peak hours compared to a conventional system, by allowing some of the vehicles to perform *express services* away from the city center. For simplicity, we only consider morning peak hours, but the same system could also be applied to evening peak hours. As a result, total passenger travel time is decreased and more passengers are served especially if some vehicle capacities are considered.

2 METHODOLOGY

In a conventional system, a number of vehicles drives back and forth between a terminal and a city center using regular routes based on a fixed timetable, even when the demand towards the city center is much larger than in the other direction. In the system we propose, it is decided beforehand whether a bus heading away from the city center, should take the regular route or the express route. Taking the express route allows to increase the frequency towards the city center. Since the system no longer has a fixed headway or a periodic timetable, the departure time of each bus, from the terminal and the city center, should also be optimized. Figure 1 gives an example of a line, its regular route according to the conventional system and its express route. The express route is determined beforehand based on the location of the stops and the road network.

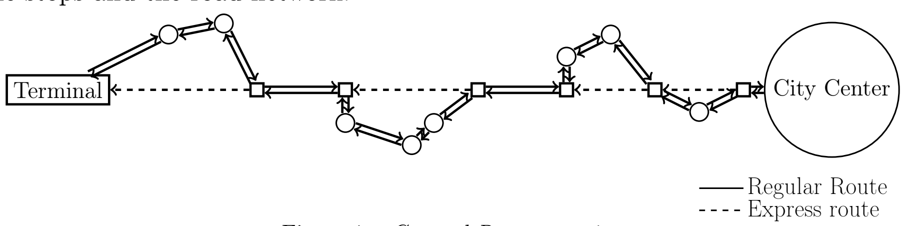

Figure 1 – *General Representation*

The demand-responsive system takes over from the conventional system at the beginning of peak hours and immediately after peak hours, it returns back to the conventional system. The following two scenarios are considered for the transition after peak hours:

1. Other vehicles are available in a depot to take over,
2. No other vehicles are available, thus the same vehicles should be available at the locations required by the regular timetable at the end of peak hours.

When optimizing this system, a fixed fleet of identical and capacitated vehicles are used, each of which performing a number of services during peak hours. When peak hours start, the buses that are on the road according to the conventional system continue their regular service at least until they reach the city center or the terminal for the first time. Then, a bus is allowed to wait before departing but an *express service* can only be performed by following the *express route* heading away from the city center. All passenger demand must be served. Based on the expected demand (distributed uniformly) from and to each stop within and outside peak hours, the objective is to minimize the total (expected) travel time of passengers traveling during peak hours measured by the sum of their in-vehicle time and waiting time.
We modeled this problem as a mixed integer quadratic program considering both scenarios for the transition after peak hours, separately. Due to the page limit, we cannot present the complete mathematical models. Instead, we focus on the objective function and the most difficult challenges and different type of constraints.
In case of constant and equal headways with no variance and uniform arrivals, the expected waiting time of passengers corresponds to half of the headway (Osuna and Newell, 1972). In our case, as a result of the service type and dwell time decisions, the headway between any two consecutive services is no longer constant. Since the headway is no longer constant, the number of passengers arriving at a stop between two consecutive services is also no longer constant and each has the expected waiting time of half of that headway, in case they can all board. This makes the total waiting time a quadratic function of the variable headway as given in Equation 1 where $w_{j,s,p}$ equals to the headway between the p^{th} and $p-1^{th}$ bus departing from stop j and serving also stop s, and $\lambda j, s$ equals to the mean demand per min from stop j to stop s during peak hours.

$$\sum_{j,s,p} \frac{w_{j,s,p}^2 \, \lambda j, s}{2} \tag{1}$$

When there is no more room in the bus, some passengers need to be left behind for the next service. Suppose $rp_{i,j,s,k,p}$ denotes the number of passengers that cannot get on the p^{th} service departing from stop j to go to stop s by bus i in its k^{th} service. It is assumed that passengers get on the buses in a first-come, first-served basis. Therefore, for each of these passengers, the expected waiting time increases by the additional time they need to wait for the next service, i.e., the headway of the next service, $w_{j,s,p+1}$. In order to correctly calculate the total travel time, the total waiting time should include an additional term where the number of passengers left behind is multiplied by the next duration of headway. However, this becomes nonlinear and not solvable by our solver. Instead, the additional waiting time for these passengers is approximated by the *normal headway, H,* of the conventional system. The additional term for the waiting time is given in Equation 2.

$$\sum_{i,j,s,k,p} rp_{i,j,s,k,p} \, H \tag{2}$$

Since the system takes over from the conventional system at the beginning of peak hours, there will probably be passengers already waiting at different stops when the peak hours start. The decisions that are made on the first services performed during peak hours will therefore affect these passengers as well. Thus, the total waiting time should include an additional term for the passengers that have arrived at their origin stops before the peak hours start and served by the first service during peak hours. Suppose $st_{j,s}^{f}$ equals the departure time of the first service during peak hours departing from stop j and serving also stop s. Then, the total waiting time for the passengers already waiting at different stops is calculated as given in Equation 3, where $\mu_{j,s}$ equals to the mean demand per min from stop j to s outside peak hours, and $tp_{j,s}$ equals to the time passed until the beginning of peak hours since the last bus departed from stop j and served also stop s before the peak hours.

$$\sum_{j,s} \frac{tp_{j,s}^2 \, \mu_{j,s}}{2} + tp_{j,s} \, \mu_{j,s} \, st_{j,s}^{f} \tag{3}$$

Due to the shorter express route on the way back, the number of passengers using an express service should be identified to reflect their gain in in-vehicle time. Other than these passengers, the in-vehicle time is the same for the conventional system. This is given below in Equation 4 where π equals the duration of the peak hours, $tt_{j,s}$ and $te_{j,s}$ equal the in-vehicle time from stop j to stop s with regular and express route, respectively, $we_{j,s,p}$ equals the headway $w_{j,s,p}$ if it is an express service and 0 otherwise, and e^f equals to 1 if the first service away from the city center is an express service and 0 otherwise.

$$\sum_{j,s} \pi\, \lambda_{j,s}\, tt_{j,s} - \sum_{j,s,p} we_{j,s,p} \lambda_{j,s}\, (tt_{j,s} - te_{j,s}) + \sum_{j,s} tp_{j,s}\, \mu_{j,s}\, (tt_{j,s} - e^f (tt_{j,s} - te_{j,s})) \tag{4}$$

Then, the objective is to minimize the total travel time as the sum of total waiting time and total in-vehicle time given in Equations 1, 2, 3, and 4. Since the order of the buses might change on the way, to be able to find the variable headway between each consecutive service from one stop to another, the service departure times should be ordered. Suppose $st_{i,j,s,k}$ denotes the service departure time of bus i from stop j and also serving stop s in its k^{th} service, then $o_{i,j,s,k,p}$ equals to 1 if bus i in its $k^t h$ service departs from stop j as the $p^t h$ service serving stop s. Equations 5a, 5b, 5c make sure that $q_{i,j,s,k,p}$ equals to $st_{i,j,s,k,p}$ if $o_{i,j,s,k,p}$ equals to 1 and 0 otherwise. Equation 5d makes sure that only one service departure time can be considered for the position of being the p^{th} service, and Equation 5e makes sure that each service departure time should be assigned to exactly one position p. Then the ordered service departure times are used to determine the headway between the p^{th} and $p-1^{th}$ service departing from stop j and serving stop s, $w_{j,s,p}$ in Equation 5f.

$$q_{i,j,s,k,p} \leq o_{i,j,s,k,p}\, M \tag{5a}$$

$$st_{i,j,s,k} - M\,(1 - o_{i,j,s,k,p}) \leq q_{i,j,s,k,p} \tag{5b}$$

$$q_{i,j,s,k,p} \leq st_{i,j,s,k} \tag{5c}$$

$$\sum_{p} o_{i,j,s,k,p} = 1 \tag{5d}$$

$$\sum_{i,k} o_{i,j,s,k,p} = 1 \tag{5e}$$

$$\sum_{i,k} q_{i,j,s,k,p} - \sum_{i,k} q_{i,j,s,k,p-1} = w_{j,s,p} \tag{5f}$$

In the rest of the model, we have several groups of constraints to make sure that everything works as intended. A group of constraints ensures that the vehicles that are on the road first continue their services until they reach the terminal or the city center for the first time. Then, based on the service type and departure time decisions, the service departure times, $st_{i,j,s,k}$, are determined for each service. However, the number of services that will be completed by each bus is unknown and also depends on these decisions. Thus, we include some constraints that allow us to tackle this issue. We also have the necessary *inventory balance type of constraints* to keep track of the number of passengers that can and cannot get on a service due to vehicle capacity. Lastly, and only for scenario 2, the final locations of the vehicles should correspond to those locations that make it possible to start running the conventional system again immediately after the peak hours.

3 RESULTS

In order to evaluate the performance of the system, three different lines are considered to design benchmark instances: S (small), M (medium), L (large) lines with 2, 8, and 14 stops and a city center, respectively. More details can be found in Table 1.

Table 1 – *Instance Characteristics*

Instance,	# of Stops		One-way driving time (min)		Peak Hours	Fleet
	Regular	Express	Regular	Express	π, (mins)	Size, m
S	1	1	10	5	30	2
M	4	4	20	10	40	2
L	7	7	30	15	60	2

We consider a demand scenario where the mean demand to travel to the city center is 20 times the mean demand to travel from the city center during peak hours and the mean demand in both directions outside peak hours. For each line, three different types of vehicles with capacities of 85, 70 and 60 are considered. The models are solved by the solver CPLEX version 12.7 with GAMS IDE on a computer equipped with a processor AMD Ryzen 7 PRO 2700U w/Radeon Vega Mobile Gfx 2.20 GHz. The optimal solutions obtained by the mathematical models for Scenario 1 and 2 are reported with the upper-bounds when the optimal solution is not found within given computational time. Since the objective function used in the model approximates the waiting time for the passengers that are left behind, the actual total travel time is also reported. Table 2 below summarizes the results.

Table 2 – *Overall Results*

				Scenario 1					Scenario 2				
Line	C (people)	CS (min)	NP-CS (people)	DRS-1 (min)	Imp-1 %	OBJ-1 (min)	CPU-1 (s)	NP-1 (people)	DRS-2 (min)	Imp-2 %	OBJ-2 (min)	CPU-2 (s)	NP-2 (people)
S	85	3156	80	2818	11%	2818	97	66	3032	4%	3032	85	66
S	70	3616	70	2818	22%	2818	-	66	3032	16%	3032	-	66
S	60	4016	60	3078	23%	3073	352	60	3123	22%	3117	124	60
Average	-	3596	-	2905	19%	2903	225	-	3062	14%	3061	105	-
M	85	3647	82	3370	8%	3370	3600	77	3531	3%	3531	3600	59
M	70	3870	70	3528	9%	3528	3600	59	3531	9%	3531	-	59
M	60	4070	60	3528	13%	3528	3600	59	3531	13%	3531	-	59
Average	-	3862		3475	10%	3475	3600	-	3531	8%	3531	3600	-
L	85	5640	85	5547	2%	5547	10800	63	5493	3%	5493	10800	63
L	70	6095	70	5531	9%	5531	10800	59	5493	10%	5493	-	63
L	60	6395	60	5531	13%	5531	-	59	6395	-	6395	-	60
Average	-	6043	-	5824	8%	5824	10800	-	5794	4%	5794	10800	-

C: Capacity, CS: Total travel time in conventional system, NP-CS: Maximum number of passengers on bus in CS, DRS-1(2): Total travel time of proposed system for scenario 1(2), Imp-1(2): Improvement of DRS-1(2) over CS, OBJ-1(2): Objective function value scenario 1(2), NP-1(2): Maximum number of passengers on bus in DRS-1(2)

According to the results given in Table 2, for all instances, the proposed system improves the total travel time when other vehicles take over the system immediately after peak hours (Scenario 1), compared to Scenario 2, where the current vehicles are required for the conventional system to take over. This is obviously expected. For those instances where the bus capacity is set to 85 people, the results of the conventional system in fact correspond to a capacity that is large enough to accommodate all passengers arriving at a stop. For the other cases where the capacity is smaller, the improvement increases further since the bus capacity is insufficient for the conventional system. However, in the proposed system, passengers accumulate less at their origin stops as a result of the higher frequency. It is also observed that the actual total travel time and the approximated objective function values are very close in all of the experiments. When the demand-responsive system operates such that no passengers are left behind, both are equal, in the other cases, the difference is rather small. Overall, compared to the conventional system, the proposed system improves the total passenger travel time about 12% and 9% on average in case of Scenario 1 and 2, respectively. It should be also noted that passengers traveling towards the city center and the express stops benefit from the system while the passengers traveling between regular stops have to wait longer. Since those passengers are low in number, the demand-responsive system still outperforms the conventional system when all the passengers are considered. Due to the computation time, to benefit more from this system, an algorithm that finds good quality of solutions within reasonable time is required for large instances.

4 CONCLUSIONS

In this study, we present a new demand-responsive system which can be operated during peak hours when passenger flows in both directions are very different. We focus on the morning peak hours where the passenger flows towards a city center are much larger than the flows in the opposite direction. Based on the expected demand and for a single line, it is decided which services can skip some stops when departing from the city center. This way, total passenger travel time improves as the frequency of service towards the city center is increased and more passengers are served despite the vehicle capacities.
The experimental results show that the demand-responsive system potentially improves the total passenger travel time up to 10% for large instances. The results also confirm that the improvement increases when other vehicles are available at the end of the peak hours (Scenario 1) and when smaller vehicles are used. In order to be able to benefit from this system to its full extend, an algorithm that finds good quality of solutions within reasonable time for larger instances is considered as future work.

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